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Temperature effect on the femto-dimensional bound states

Scientific research paper

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ABSTRACT

The bound state of exotic gluons with a thermal background in the framework of the projective unitary representation in the physics model at high energy interactions has been investigated. The ground and excited states of gluonic systems to describe the temperature effect of the mass spectrum have been defined. The problem of calculating the mass spectrum of coupled states based on the QFT in its broadest sense is a technique, to detect and obtain reasonable objectives and goals. The aim of this paper is to present the possible use of symplectic space in Femto-dimensional bound states at a finite temperature within strong interaction. The results can be used for describing nonzero and zero temperature mass spectra of mesons and multigluon-bound states. The problem of determining coupled state masses based on the polarization function is examined in detail. The Hamiltonian of the interaction and the structure of the bound state with the Schrödinger equation within the color-confining potential and Debye mass at finite temperature is presented and then the mass spectrum of a glueball at high energy interaction is determined. The mass spectrum of a two-gluon coupled state is calculated. The results are presented in the figures and table.

1 Introduction

The bound state, with two or three gluons, gluon-quarks, quark-quarks has had a great impact in physics and also has had interesting subjects in particle physics, exotic bound state physics, astrophysics, and quantum field theory. The actuality and objectivity of exotic and quasi-exotic states have been the topic of severe dependence among physicists in recent activities. These quasi-exotic bound states of hadrons are considered to be in two, three, four, and more multiparticle states such as quarkonium, glueball, and di-meson. A well-known technique of considering the exotic relativistic states as a strong or high energy collision was raised in 1987-88, where the asymptotic behavior of n-point Green's function at high energy limit is acquired in the exponentiated model using the large asymptotic limit of $x \to \infty$ from the Feynman functional path integral (FFPI) in the external vector field [1,2]. The technique is applied to the basic fundamental model of quantum chromodynamics and quantum electrodynamics.

In this approach the properties of the hadronic vacuum polarization function in closed-loop through the FFPI is offered; The FFPI cannot be calculated as usual; in this case, we have to simplify physical assumptions that were made [1-3]. A different form and method of calculating FFPI and defining double-gluon (glueball)

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and tri-gluon (pomerons), gluon-quark mass at finite temperature are defined based on the quantum field theory approach [4] .

The finite temperature-dependent in a double-gluon bound system at high energy and strong interaction is a very important subject in theoretical and experimental physics. The temperature relation for the color-confining potential describes the new characteristic of the double-gluon bound state (the glueball is a hypothetical composite particle) [5,6], as we know the state of two particles (bosonic or fermionic) can be described by symmetric and anti-symmetric states:

$$|a \otimes b\rangle_{boson} =$$

$$2^{-1/2}(|a_1 \otimes b\rangle_2 + |a_2 \otimes b_1\rangle)\,,$$

and

$$|a \otimes b\rangle_{fermion} =$$

$$2^{-1/2}(|a_1 \otimes b_2\rangle - |a_2 \otimes b_1\rangle).$$

Now we try to describe interaction in temperature-dependent by the modified radial Schrödinger equation (MRSE) and using the analytic method based on the behavior of the correlation function of gluons at finite temperature in the strong field. The determination of mass spectrum of glueball and pomeron systems is suggested within, this idea and also, the binding energy, eigenvalue energy, and mass spectra and eigenenergy of the bound states. The two-gluon /three-gluon states include those which do not fit in the well-known states (quark-gluon condense matter at high temperature). They include in the multigluon states. The gluonic restricted (surrounded) system is a multibody state that has been studied in the framework of QCD confinement and methods such as the Gaussian expansion method, the quantum chromodynamics sum rules, and the Lattice quantum dynamics. Therefore, based on the quantum chromodynamics models and theories we can determine that the n-gluon system is one of the most crucial states which makes it possible to create restricted and constrained states of vector bosons at high finite temperatures.

In this article, we show that the mass spectrum of the n-gluon state is extremely higher than predicted in theory (that did not include temperature effect in calculations). Mathcad 15.0 M050 program and Excel 2019 software were used to assess parameters, and calculate their values. All theoretical, mathematical, data processing and outcomes were formed by the author. Therefore, this study leads to greater awareness and interest in theoretical and experimental interpretations. We study exotic bound systems as double-gluon at finite temperature using the extreme limit properties (at $x \to \infty$) of Gaussian processes

$$(\Psi_{min} = N_{min} e^{-\frac{m\omega}{2\hbar}r^2}),$$

where

$$\int_{-\infty}^{\infty} dr \Psi^*_{min} \Psi_{min} = 1 \Rightarrow$$

$$\Psi_{min}(r) = (\frac{m\omega}{\pi\hbar})^{1/4} e^{-\frac{m\omega}{2\hbar}r^2}),$$

of the dynamical correlation functions of the corresponding vector field currents for the determination of the mass spectrum and eigenenergy values in the lower and higher state of a double-gluon system with the strong interaction confinement potential [2,3]. We also get a new relativistic-temperature correction to the relativistic distribution of the effective mass of the double-gluon system (constituent mass). The mass spectrum and the relative distribution mass are determined from the MRSE [7-9]:

$$\hat{H}R(r) = MR(r) = E_\ell(\mu)R(r), \tag{1}$$

The effective mass of the double-gluon system by modifying relativistic correction is defined. Now we describe the method: as we know, in the quantum field, the mass of the coupled gluons is presented by the Gaussian shape of the correlated-current function and the exact orbital quantum numbers. The statistical correlation, in the Green function condition, is presented. It is defined as FFPI and lets the necessary asymptotic properties of Gaussian processes limit to be allocated, then one can carry out the average value of the external gauge field. In nonrelativistic quantum mechanics, the resulting image of presentation is similar to the FFPI. Hence, the mass of coupled gluon-gluon determines the polarization function (statistical correlation) $\Pi(r)$ and the Green's function $G(r)$ [2]. We know, the current of the charged scalar field $J(r) = R^{+}(r)R^{-}(r)$ and we overlook the destruction channel, then it is appropriate to represent the correlators as a product of the Green's functions of the scalar charged field, after averaging over the fluctuating external field $A_\mu(r)$. Now, one can define the Green's function of the scalar charged particles by

$$\left[\left(i\partial_\mu + \frac{g}{c\hbar}A_\mu\,(r_1 - r_2)\right)^2 + \frac{c^2m^2}{\hbar^2}\right]G\left(r_1, r_2\middle|A_\mu\right) = \delta(r_1 - r_2), \qquad (2)$$

where $A_\mu\,(r)$ is the external gauge field and in the other sections we consider $A_\mu\,(r) = 0$, g is coupling constant of interaction, m is the mass of the scalar particle, δ is Dirac's delta function, and the Green function is defined as below, where H is the Hamiltonian, θ is the time function of Hamiltonian

$$G(r_1, r_2, t_1, t_2) = \theta(t_2 - t_1)\exp\left(\frac{-i}{\hbar}H(t_2 - t_1)\right)\delta(t_2 - t_1),$$

which satisfies this type of differential equation

$$\left[H(t_2 - t_1) - \frac{i}{\hbar}\frac{\partial}{\partial t}\right]G(r_1, r_2, t_1, t_2) = -i\hbar\theta\delta(r_2 - r_1)\delta(t_2 - t_1).$$

We have to define a new form for the Green function based on FFPI. Hence, solution of the Green's function with $t_2 > t_1$ is described in the FFPI form (for full detail see [2, 10]):

$$G(r_1, r_2, t_1, t_2) = \exp\left(\frac{-i}{\hbar}H(t_2 - t_1)\right)\delta(r_2 - r_1) = T_r \exp\left[-i\int_{t_1}^{t_2} d\tau\left(\frac{-1}{2m}\left(\frac{\partial}{\partial r(\tau)}\right)^2 + V(r(\tau))H(t_2 - t_1)\right)\right]\delta(r_2 - r_1).$$

Here, τ is the time parameter and T_r is the time-ordering symbol and is used for time-ordering of operators $r(\tau)$, $\frac{\partial}{\partial r(\tau)}$. Now using the well-known integral representation of the form

$$\exp\left(-\frac{i}{2}\int_0^\alpha d\tau\left(\frac{\partial}{\partial r(\tau)}\right)^2\right) = C\int dv\exp\left[\left\{-\frac{i}{2}\int_0^\alpha d\tau v^2(\tau) + \right\} + \exp\left\{\int_0^\alpha d\tau v\,(\tau)\left(\frac{\partial}{\partial r(\tau)}\right)\right\}\right],$$

the Green function in the external gauge field can be presented in the form of the following (full detail can define in Ref. [10] pages 138-142):

$$G\left(r_1, r_2\middle|A_\mu\right) = \int_0^\infty d\alpha\,\exp\{-m^2\alpha\}\exp\left\{-\int_0^1 d\tau\left(\alpha i\partial_\mu + \alpha g A_\mu\left(r_1(\tau)\right)\right)^2\right\}\delta(r_1 - r_2), \qquad (3)$$

where $(\hbar = c = 1)$, $r = r_1 - r_2$,

$$J\,(r) \cong \exp\left(-xE_\ell(\mu)\right),$$

$$\Pi(r) \cong \exp\left(-M\sqrt{r^2}\right),$$

and also, we know the coupled gluon-gluon mass spectrum should explain in relativistic-quantum theory by polarization function $\Pi(r)$ (for full detail see [2,10]):

$$M = -\lim_{|r|\to\infty}\frac{ln\Pi(r)}{|r|} = -\lim_{|r|\to\infty}\frac{ln\langle G_1(r)|G_2(r)\rangle}{|r|}.$$

After simplifying the equation, the mass spectrum reads $(i = 1,2)$:

$$\frac{\partial M}{\partial \mu_i} = \frac{\partial}{\partial \mu_i}\left(\left(\frac{\mu_2 m_1^2 + \mu_1 m_2^2}{2\mu_1\mu_2}\right) + 0.5(\mu_1 + \mu_2) + E_\ell(\mu)\right) = 0. \qquad (5)$$

Therefore, the gluon-gluon system with the zero-rest mass:

$$m_1 = m_2 = m_g = 0$$

and the constituent mass of components in the bounding system

$$\mu_1 = \mu_2 = \mu_g. \tag{6}$$

Hence, the glueball bound state mass in the stationary state reads:

$$\frac{\partial M}{\partial \mu_1} = \frac{\partial}{\partial \mu_1}\left(\frac{\mu_2 m_1^2 + \mu_1 m_2^2}{2\mu_1\mu_2} + \frac{\mu_1}{2} + E_\ell(\mu)\right) = 0,$$

$$\frac{\partial M}{\partial \mu_2} = \frac{\partial}{\partial \mu_2}\left(\frac{\mu_2 m_1^2 + \mu_1 m_2^2}{2\mu_1\mu_2} + \frac{\mu_2}{2} + E_\ell(\mu)\right) = 0$$

$$\frac{1}{\mu} = \frac{\mu_1 + \mu_2}{\mu_1\mu_2} = \frac{2}{\mu_g}, \tag{7}$$

where one can determine parameter μ_g as follows [10]:

$$\mu_g = \frac{\partial M}{\partial \mu_g} = 0 \Rightarrow$$

$$\mu_g = \sqrt{m_g^2 - 2\mu^2 \frac{dE_\ell}{d\mu}} = \sqrt{-2\mu^2 \frac{dE_\ell}{d\mu}}. \tag{8}$$

2 Theoretical formalism

2.1 Quasi-exotic bound state

Double-gluon bound state is a quasi-exotic hadronic state explained by the linear color-confining potential for gluons at the range of strong coupling constants $\sigma(r)$, and at finite temperature. The glueball-bound state at a finite temperature [8,9] using the MRSE is specified, which enabled us to describe and define glueball specifications [11-13]. Therefore, in the present work, we consider a system consisting of the main state and the first radial lowly and highly excited states of the double-gluon bound state. Such a method should be considered as a good approximation to the more excited states. Analyses results show that the colored constituent particles inside the glueball at finite temperature (i.e. gluons) can describe the new characteristic of bound state mass. This is a good approximation for defining characteristics of gluonic systems in strong interactions. In the framework of this study, we determined the quasi-exotic bound state mass in the color-confining potential and/or temperature-dependent [16]:

$$V(r) = \sigma(r)r, \sigma(r) = \frac{4}{9}\sigma_g.$$

$$V(r,T) = \sigma(r,T)r. \tag{9}$$

at the finite temperature according to the projective unitary representation (PUR) [10-12]. In this case, using the modified radial relativistic Schrödinger equation, we have

$$\widehat{H}R(r) = MR(r) = E_\ell(\mu)R(r).$$

Now, we explain the relativistic effect on the bound states using explanations I and II (defined based on the saddle point method or the method of steepest [10]) according to Eq. (7)

$$I. \sqrt{m^2 + \hat{p}_r^2} = m\sqrt{1 + \frac{\hat{p}_r^2}{m^2}} \approx$$

$$\approx m + \frac{\hat{p}_r^2}{2m} - \frac{\hat{p}_r^4}{8m^3} + \cdots,$$

$$II. \sqrt{m^2 + \hat{p}_r^2} \approx \min_\mu \frac{1}{2}\left(\mu + \frac{m^2 + \hat{p}_r^2}{\mu}\right),$$

and then the modified radial relativistic Schrödinger equation reads

$$MR(r) = \left(\sqrt{m_1^2 + \hat{p}_r^2} + \sqrt{m_2^2 + \hat{p}_r^2} + V(r,T)\right) R(r)$$

$$\Rightarrow$$

$$\left(m_1 + \frac{\hat{p}_r^2}{2m_1} - \frac{\hat{p}_r^4}{8{m_1}^3} + \cdots\right) R(r) +$$

$$\left(m_2 + \frac{\hat{p}_r^2}{2m_2} - \frac{\hat{p}_r^4}{8{m_2}^3} + \cdots\right) R(r) +$$

$$V(r,T)R(r) - MR(r) = 0, \tag{10}$$

where $|\hat{p}_1^2| = |\hat{p}_2^2| = |\hat{p}_r^2$.

Two approximate methods usually are used to predict the structure of the bound states and then the Hamiltonian of glueball bound state is given by ($\hbar = c = 1$)

$$\left[\frac{1}{2}\left(\mu_1 + \frac{m_1^2 + \hat{p}_r^2}{\mu_1}\right) + \frac{1}{2}\left(\mu_2 + \frac{m_2^2 + \hat{p}_r^2}{\mu_2}\right) + \sigma(r,T)r\right] R(r) = MR(r)$$

$$= E_\ell(\mu) R(r) \Rightarrow$$

$$\left[\frac{\hat{p}_r^2}{\mu_g} + \sigma(r,T)r\right] R(r) = \left(M - 2\mu_g\right) R(r)$$

$$= \left(M - 4\mu\right) R(r), \tag{11}$$

and

$$M = 2\sqrt{-2\mu^2 \frac{dE_\ell}{d\mu} + \mu\, \frac{dE_\ell}{d\mu}} + E_\ell,$$

$$E_\ell(\mu) = M - 2\mu_g, \tag{12}$$

where μ_g is the constituent mass of components (glueballs) in the bounding system and μ is the effective inertial mass that is determined and presented in the following paragraph.

The MRSE in the n-dimensional space can present as a first and second-order differential equation:

$$\frac{1}{2r^{n-1}} \frac{d}{dr}\left(r^{n-1} \frac{d}{dr}\right) R(r) -$$

$$-\frac{\ell(\ell + n - 2)}{2r^2} R(r) - \mu\sigma(r,T) r R(r)$$

$$+ \mu\left(M - 2\mu_g\right) R(r) = 0. \tag{13}$$

We know that the nonzero temperature described by the exponential function depends on the Debye mass or the thermal mass [1]: $m_D(T) \cong gT$, where g describes strong constant of interaction at $0 \prec T < 175\text{MeV}$, ($T_c \approx 175\text{MeV}$) using the approximation limit to the nonzero temperature for the color charge confinement:

$$\sigma(r,T) \approx \frac{\sigma(T)}{m_D(T) r}\left(1 - e^{-m_D(T) r}\right) \approx$$

$$\sigma(T) - \frac{\sigma(T)}{2!} m_D(T) r + \frac{\sigma(T)}{3!} (m_D(T) r)^2. \tag{14}$$

We determine mass spectrum at finite temperature $0 \prec T < 175\text{MeV}$, in the zero-order of Debye mass approximation. Thus, at finite temperature Eq. (14) reads:

$$\sigma(r,T) = \sqrt{1 - \left(\frac{T}{T_c}\right)^2}\, \sigma(r,0) =$$

$$= \sqrt{1 - \left(\frac{T}{T_c}\right)^2}\, \sigma(r), \tag{14*}$$

where $\sigma(r,0) = \sigma(r)$.

2.2 The aim of the research

In this article theoretically, the double-gluon system as a bound state solution by Schrödinger equation in color-confining potential at finite temperature has been described. For this opinion, the projective unitary method (PUR) of the Schrödinger equation is used. The behavior of the gluons bound state close to the deconfinement temperature is very important in the high-energy interacting environment. The results of calculation using different models usually have uncertain values and we cannot predict the exact value of mass. We have to predict the mass spectrum using the relativistic behavior of gluons. Therefore, we present the bound state mass spectrum at finite temperature based on quantum field theory and determine the relationship between mass spectrum and temperature. The temperature-dependent modified RSE is considered by using the PUR representation [1,2]. The color-confining potential in the Schrödinger equation is used. The bound state eigenvalue and mass spectrum of double-gluon are presented in the form of an analytical method and numerical results are defined at zero and finite temperatures.

3 Temperature dependence of MRSE

Radial Schrödinger equation of the system with color-confining interaction's potential between gluons reads:

$$\left\{\frac{\hat{p}_r^2}{2\mu}+\sigma(r,T)r-E_\ell(\mu,T)\right\}R(r) = 0, \qquad (15)$$

and using the Debye mass, equation reads

$$\left\{\frac{\hat{p}_r^2}{2\mu}+\left(\frac{\sigma(T)}{m_D(T)r}\left(1-e^{-m_D(T)r}\right)\right)r-E_\ell(\mu,T)\right\}R(r) = 0. \qquad (16)$$

In quantum field theory for the ground and vacuum states the systems at finite temperature are described by an infinite number of oscillators that keep their oscillating character in interactions. To use the quantum field methods, we have to change variables in equation (4) for the linear interaction terms of color-confining by replacing a new variable [2, 14] in the $\mathcal{D}$-dimensional axillary space. Now, according to the high momentum asymptotic form of Gaussian type ($r\to\infty$): $\hat{r}=\hat{q}^{2\rho}$ and $R(\hat{r})\to R(\hat{r})=q^{2\rho\ell}R(q^2)$, where ρ is a parameter to be determined; in the color-confining potential for a double-gluon system, such a modification is performed by variational parameter ρ, the glueball wave function becomes an oscillator one. The radial Laplacian operator in the $\mathcal{D}$-dimensional axillary space [4]: $\mathcal{D}=4\rho\ell+2\rho+2$, ($\ell$ is the angular momentum quantum number) can define using the radial Laplacian operator in the n-dimensional space:

$$\Delta_r=\frac{d^2}{dr^2}+\frac{n-1}{r}\frac{d}{dr}\to$$

$$\Delta_q=\frac{d^2}{dq^2}+\frac{\mathcal{D}-1}{q}\frac{d}{dq}.$$

Here, the wave function should have the Gaussian type solution for large distances and we apply the PUR [2] variables from Eq. (8) to the Hamiltonian (Eq. (7)). Equation (7) in a new auxiliary space ($q^{2\rho}$) is obtained:

$$\left\{\frac{\hat{p}_q^2}{2}+4\mu\rho^2\hat{q}^{4\rho-2}\left(\sigma(T)\hat{q}^{2\rho}-\frac{\sigma(T)}{2}m_D(T)\hat{q}^{4\rho}+\frac{\sigma(T)}{3!}m_D^2(T)\hat{q}^{6\rho}+\cdots-E_\ell\right)\right\}R(\hat{q}^2)=0,$$

$$\left\{\frac{\hat{p}_q^2}{2}+4\mu\rho^2\sigma(T)\hat{q}^{6\rho-2}-\frac{\sigma(T)}{2}m_D\hat{q}^{8\rho-2}+\frac{\sigma(T)}{3!}m_D{}^2\hat{q}^{10\rho-2}+\cdots-4\mu\rho^2\hat{q}^{4\rho-2}E_\ell\right\}R(\hat{q}^2)=0, \qquad (17)$$

where

$$\hat{p}_q^2 = \frac{(2+2\rho+4\rho\ell)}{2}\omega_\ell +:\hat{p}_q^2:. \quad (18)$$

and

$$\hat{q}^2 = \frac{1}{2\omega_\ell}(2+2\rho+4\rho\ell)+:\hat{q}^2:.$$

$$\hat{q}^4 = \frac{1}{4\omega_\ell^2}(2+2\rho+4\rho\ell)\times$$

$$(4+2\rho+4\rho\ell)+:\hat{q}^4:. \quad (18^*)$$

We can determine mass spectrum with the first, second, and third-order of Debye mass from Eq. (17) by using:

$$\hat{q}^6 = \frac{1}{8\omega_\ell^3}(2+2\rho+4\rho\ell)\times$$

$$\times(4+2\rho+4\rho\ell)\times(6+2\rho+4\rho\ell)+:\hat{q}^6:$$

$$\Rightarrow \hat{q}^6 = \frac{L_\ell}{8\omega_\ell^3}+:\hat{q}^6:.$$

and

$$\hat{q}^{2m} = \frac{1}{\omega_\ell^m}\frac{\Gamma\left(\frac{2+2\rho+4\rho\ell}{2}+m\right)}{\Gamma\left(\frac{2+2\rho+4\rho\ell}{2}\right)}+:\hat{q}^{2m}:, \quad (19)$$

$\hat{q}, \hat{p}_q$ are canonical operators and can be presented by $\hat{a}^+$ and $\hat{a}^-$ operators [4]:

$$\hat{a}^- = \sum_k \langle q|p\rangle \hat{a}^- = 2^{-1/2}(\hat{Q}+i\hat{P}_q),$$

$$\hat{a}^+ = \sum_k \hat{a}^+\langle p|q\rangle = 2^{-1/2}(\hat{Q}-i\hat{P}_q), \quad (20)$$

where

$$\hat{q} = \left[\frac{m\omega}{\hbar}\right]^{1/2}\hat{Q} = \left[\frac{2m\omega}{\hbar}\right]^{1/2}(\hat{a}^+ + \hat{a}),$$

$$\hat{p}_q = [\hbar m\omega]^{1/2}\hat{P}_q = i\left[\frac{m\omega}{2\hbar}\right]^{1/2}(\hat{a}^+ - \hat{a}).$$

The canonical variables as Wick ordering based on the PUR condition are obtained in Eqs. (19) and (20) (see [2,4] in more detail). According to the PUR conditions the interaction Hamiltonian contains all non-square parts of the term :∗: (a condition in Wick ordering) and then we can find the renormalization of the bound state parameters like wave function which let us introduce the zero approximation in the PUR and then find the eigenvalue of the ground state energy $\varepsilon_0(E_\ell)$ [14]:

$$\begin{aligned}\varepsilon_0(E_\ell) = {} & \frac{\hat{p}_q^2}{2} + 4\mu\rho^2\sigma(T)\hat{q}^{6\rho-2} \\ & - 2\mu\rho^2\sigma(T)m_D\hat{q}^{8\rho-2} \\ & + \mu\rho^2\frac{\sigma(T)}{3!}{m_D}^2\hat{q}^{10\rho-2} + \cdots \\ & - 4\mu\rho^2\hat{q}^{4\rho-2}E_\ell = 0, \end{aligned} \quad (21)$$

Thus Eq. (16) according to $R(r) = \hat{q}^{2^{2\rho\ell}}R(\hat{q}^2)$, can be defined based on the energy eigenvalue and by making it least in the form of the zeroth approximation (see [12] in more detail) with the zero-order of Debye mass approximation using equation (14*):

$$\sigma(r,T) \approx \sigma(T)$$

then Eq. (21) in strong in ultra-strong interaction, based on our supposed ($\rho \cong 1$) reads:

$$\begin{aligned}\varepsilon_0(E_\ell) = {} & \frac{(2+2\rho+4\rho\ell)}{4}\omega_\ell \\ & + \frac{\mu\sigma(T)}{\omega_\ell^2}(2+2\rho+4\rho\ell)\times(4+2\rho \\ & + 4\rho\ell)) - 2\mu E_\ell\frac{(2+2\rho+4\rho\ell)}{\omega_\ell} \\ & = 0, \end{aligned} \quad (22)$$

Then rewrite Eq. (22) as follows

$$\varepsilon_0(E_\ell, T) = X(T,\omega_\ell) - E_\ell Y(T,\omega_\ell) = 0.$$

and then based on PUR conditions [2], we find

$$\varepsilon_0(E_\ell, T) = 0, \qquad \frac{d\varepsilon_0(E_\ell, T)}{d\omega_\ell} = 0, \quad (23)$$

Now, we can determine the extreme eigenenergy's value of the ground state of the bound system as a result of the zeroth-order approximation (see [12] in more detail). Therefore,

$$E_\ell(T) = \frac{1}{4\mu}\omega_\ell^2 + \frac{\sigma(T)}{2\mu\omega_\ell} \times (4 + 2\rho + 4\rho\ell), \qquad (24)$$

and based on Eq. (23) we define the equation for the oscillator frequency as

$$\omega_\ell^3 - 2(8 + 2\rho + 4\rho\ell)\mu\sigma(T) = 0, \qquad (25)$$

and, the oscillator frequency reads

$$\omega_\ell = \sqrt[3]{2(8 + 2\rho + 4\rho\ell)\mu\sigma(T)} =$$

$$\sqrt[3]{\sqrt{m_g^2 - 2\mu^2 \frac{dE_\ell}{d\mu}} \times 2(8 + 2\rho + 4\rho\ell).\sigma(T)}, \qquad (25^*)$$

From Eqs. (24)-(25), after some representation eigenenergy reads:

$$E_\ell(T) = \frac{3}{2}\sqrt[3]{\frac{(3 + 2\ell)^2}{4\mu}\sigma^2(T)}. \qquad (26)$$

Thus, using all formulas in Eqs. (6) and (24), then the mass spectrum of the predicted bound state can be defined and specified as a function of μ and T. Now in the next paragraph, we utilize results to distinguish the specifications of the glueball constrained mode.

4 The spectrum of glueball mass

This approach introduces PUR to the energy and mass spectra of a bound state. Therefore, to estimate the accuracy of the PUR in particle physics we compare it to the results of other calculations. In this case, the interaction potential is of a confinement potential, and from Eq. (23) with constituent mass that is given from Eq. (6), we obtain the mass and binding energy spectrum [15]. Next, we determine the bound parameters composed of double-gluon or the scalar glueball (gluonium) in the $J^{PC} = 0^{++}$, but in the table, we have included some of the other glueball states with a different quantum number like $0^{--}, 0^{-+}, 2^{++}$.

Glueball bound state's mass and binding energy at finite temperature were determined by several authors using different models based on phenomenological potential models and field approaches. The following limits on the glueball system are given $0 \prec T < 175\text{MeV}$, $T_c \cong 175\text{MeV}$[16], where T_c is the confinement temperature, and temperature constant parameter $\sigma_g \cong 0.18\text{GeV}^2$ [17] is obtained from [18] for strong interaction:

$$V(r,T) = \frac{4}{9}\sqrt{1 - \left(\frac{T}{T_c}\right)^2}\,\sigma_g r, \qquad (27)$$

and coupling constant in the range $\sigma(r) \cong 0.405\text{GeV}^2$[16,17], in the color-confining potential $V(r) = \sigma(r)r$. With the values of the zero-rest mass and angular momentum quantum number, we obtained the constituent mass of the gluon and mass spectrum at finite temperature in the glueball-bound states. The defined numeric results of the mass spectrum and the constituent gluon mass for bound state at finite temperature are shown in Table 1. For more description and relevance of this research, we plot two figures. In Fig 1. The variation of double-gluon bound state mass with temperature is shown for the main, lower, and higher energy states.

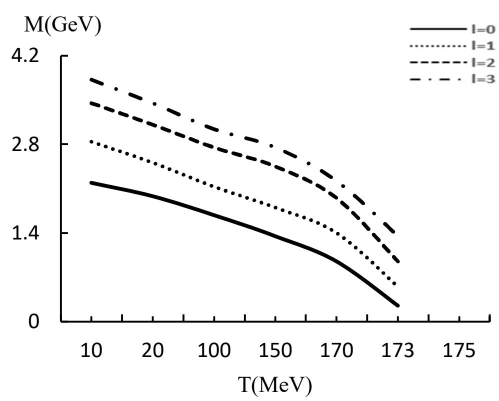

Figure 1. The glueball mass spectrum as a function of the temperature ($T_c \cong 175\text{MeV}$) in the main, lower, and higher states (ℓ = 0,1,2,3).

In Figure 2, the mass spectrum of glueball with different orbital quantum numbers are plotted: without potential interaction related to the temperature $V(r)$, and with potential interaction related to the temperature $V(r,T)$ at $T = 0.01\text{MeV}$, that can be $T \approx 0$. The mass spectrum of glueball without potential interaction related to the temperature $V(r)$ [16], and with potential interaction related to the temperature without spins interaction $V(r.T)$ can be compared with data in the references [18,19,20,21,22] that presents potential interaction related to the temperature and/or the spin's interaction, one gluon exchange, and/or non-perturbative behavior.

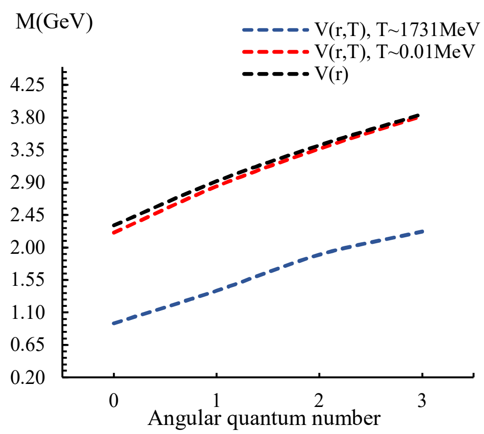

Figure 2. The glueball mass spectrum as a function of the angular momentum quantum number in the color-confining potential $V(r)$, and $V(r,T)$ at $T = 0.01MeV$, ($T_c \cong 175MeV$).

The Hamiltonian, one gluon exchange and non-perturbative behavior, based on these theoretical results, can more decrease the glueball bound state mass and the constituent gluon mass at finite high-temperature $T < T_c$.

5 Gluonium wave function at finite temperature

Now, we try to formularize the double-gluon lowest-energy state (ground state) wave function. The $\mathcal{D}$ -dimensional wave function of the ground state in terms of the PUR model has the form [2]:

$$|\mathcal{R}_0\rangle = \left(\frac{\omega_0}{\pi}\right)^{\frac{4\rho\ell+2\rho+2}{4}} \exp\left(-\frac{\omega_0}{2}q^2\right) \quad , \qquad (28)$$

In the case under our consideration with the axillary space $\mathcal{D} = 4$ and the angular momentum quantum number $\ell = 0$, we have the pure oscillator frequency

$$\omega_0 = \sqrt[3]{\beta\sigma(T)},$$

and then the ground state wave function can be defined as

$$|\mathcal{R}_0\rangle = \left(\frac{\sqrt[3]{\beta\sigma(T)}}{\pi}\right) \exp\left(-\frac{\mu\sqrt[3]{\beta\sigma(T)}}{2}q^2\right)' \qquad (29)$$

where satisfies the well-known conditions:

$\langle\mathcal{R}_0|\mathcal{R}_0\rangle = 1$, and $\hat{a}^-|\Psi\mathcal{R}_0\rangle = 0$.

All higher states with the angular momentum quantum number $\ell \neq 0$ can be determined by $|\mathcal{R}_\ell\rangle = \left(\hat{a}^+\hat{a}^+\right)^\ell|\mathcal{R}_0\rangle$. The wave function of glueball at zero and finite temperature with $\ell = 0,1,2,3$ are determined (without spin-orbit interactions). in Figure 3, the probability density of radial wave functions of glueball bound state with $V(r.T)$ at $\left(T_3 > T_2 > T_1\right)MeV$ are plotted. The curve is presented as a Gaussian type distribution for wave function by fitting points. The plot gives us the relativistic amplitude and shows that the amplitude of $V(r)$ is higher than the others in the ground state, as we can see in Figure 3,

$$|\mathcal{R}_{0T_1}| > |\mathcal{R}_{0T_2}| > |\mathcal{R}_{0T_3}|,$$

and at the critical deconfinement temperature $T_c \geq 175MeV$, glueball bound state is vanished and destroyed.

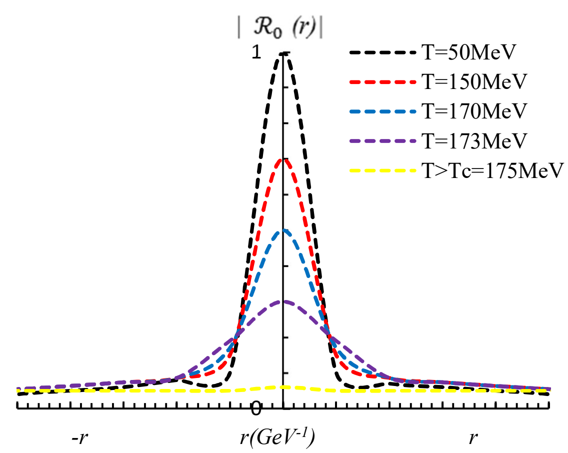

Figure 3. The glueball ground-state wave function ($\ell = 0, T_5 > \cdots T_2 > T_1$) in the color-confining potential, the black line presents $V(r)$ [16] and the other lines present $V(r,T)$ results of this work.

With increasing temperature, the wave function of glueball vanishes and the bound state of Glueball gets destroyed:

$(T \to T_c \geq 175MeV \Longrightarrow |\mathcal{R}_0| \to 0)$.

Table 1. Mass spectrum and constituent mass of double-gluon system at finite temperature. The values are in terms of GeV.

	$V(r)=\sigma(r)r$ $\rho\cong 0.57$	$V(r)=\sigma(r)r$ $\rho\cong 1$ THIS WORK	$V(r.T)=\left(\sqrt{1-\left(\frac{T}{T_c}\right)^2}\sigma(r)\right)r$ $\rho\cong 1$ THIS WORK			Exp	Theor.
T	-	-	0.01	0.02	0.10	-	-
	$\ell=0$						
M	2.1116 Ref[16]	2.2045	2.2027	2.1973	1.9971	2.11 Ref[23] 2.32(0^{++}) Ref[24] 2.33(2^{++}) Ref[30]	1.98 Ref[25] 2.69 Ref[26]
μ_g	0.5279 Ref[16]	0.5511	0.5506	0.5493	0.4992	-	-
	$\ell=1$						
M	2.7722 Ref[16]	2.8460	2.8437	2.8367	2.5782	-	2.95(0^{-}) Ref[27]
μ_g	0.6930 Ref[16]	0.7115	0.7109	0.7091	0.6445	-	-
	$\ell=2$						
M	3.3044 Ref[16]	3.3674	3.3647	3.3564	3.0506	-	3.152(1^{++}) Ref[30] 3.39(0^{++}) Ref[27] 3.47(2^{++}) Ref[27]
μ_g	0.8261 Ref[16]	0.8418	0.8411	0.8391	0.7626	-	-
	$\ell=3$						
M	3.7625 Ref[16]	3.8183	3.8152	3.8058	3.4590	-	3.95(0^{-}) Ref[27] 3.99(3^{-+}) Ref[27] 3.90(0^{-+}) Ref[29]
μ_g	0.9406 Ref[16]	0.9545	0.9538	0.9514	0.8647	-	-

6 Discussion

The double-gluon constrained and restricted state solution of the RSE at limited temperature with color-confining potential under the projective unitary representation is defined. The analytical results for the temperature dependence are calculated. The relativistic correction to the mass is calculated as we have shown the relativistic mass relation with temperature in high-energy interactions and described that glueball mass spectrum at finite temperature with the relativistic correction is heavier than without correction. For determining temperature dependence, we used the nonzero temperature as an exponential function and modified the Debye mass. Results have shown that the nonzero and finite temperature mass spectra of glueball, can be described as the new characteristic at relativistic and ultrarealistic and also at the finite high temperatures. Based on the results we can deduce that the outcomes and consequences of this theoretical study are expected to determine new expectancy for theoretical description on the double-gluon system, because of the good and comparable results with other theoretical works. The obtained theoretical data can be useful in new research and can open new perspectives to determining the new characteristics of exotic systems. We have calculated temperature dependence on the relativistic correction to the mass of glueball in Table 1. The relativistic correction to mass and temperature-dependent will be increased by increasing orbital quantum number ℓ as shown in Table 1. But at a very high-temperature $T \to T_c = 175MeV$, we predict that based on Eqs. (15) and (16), the relation between temperature and relativistic mass of bound state have to change the main characteristic i.e. the double-gluon bound state cannot exist.

7 Conclusions

Following the discussion for the projective unitary representation method, we have studied the bound state properties of the double-gluon system at finite temperatures. We have calculated the constituent mass of gluon based on relativistic correction to the mass. In the above calculations, we have, both analytically and numerically found:

1- The relativistic behavior of constituent mass μ_g of gluon appears and in the glueball-bound state with the fixed orbital quantum number ℓ, it decreases with increasing temperature and above the critical deconfined temperature, the bound state is destroyed and gluon will be free. The relativistic correction to the constituent mas μ_g of gluon increase with increasing orbital quantum number ℓ and with the fixed temperature T.

2- The double-gluon bound state mass M decreases with increasing temperature $T < T_c = 175MeV$.

3- The constituent gluon mass μ_g decreases with temperature and above the critical temperature will be in quark-gluon plasma state.

4- At high temperatures $T \to T_c > 175MeV$ the exotic double-gluon bound state melts into the free gluons, i.e. gluonic plasma as a new phase environment. In real conditions the exotic double-gluon bound state melts into the free gluons, i.e. gluonic plasma at ultra-high temperature.

5- The amplitude of the relativistic correction to the mass of glueball bound state at $T \approx 0$ is larger than other temperatures above zero.

6- The mass spectrum and constituent mass of the glueball system increase with increasing quantum orbital number and temperature (below the critical deconfined point) $0 < T < T_c = 175MeV$.

Conflicts of interest

The Authors have no conflicts of interest.

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