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Thermal entanglement in spin chain with XX, XY, and XZ Heisenberg interactions

Scientific research paper

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ABSTRACT

In this paper, the quantum correlation of a 2-spin Heisenberg model system with *Dzyaloshinskii-Moriya* interaction in presence of an external magnetic field is studied. Three types of exchange interactions, namely, XX, XY, and XZ are considered. The thermal entanglement of these systems is investigated under the change of the external magnetic field. We found that the temperature behavior of the three models is similar. Moreover, we study the magnetic field dependence of negativity in three models and obtain the critical magnetic field.

1 Introduction

Entanglement is a basic characteristic of quantum physics which has attracted attention in recent decades as a non- classical correlation in the quantum systems [1,2,3]. Quantum entanglement has an important role in various fields of quantum information, quantum computation, quantum teleportation, quantum communication, and superdense coding [4,5,6]. In the solid state, the Heisenberg spin systems have been devoted as a suitable system to study the entanglement properties of the quantum systems. Spins have been recognized as candidates for quantum solid state research [7,8]. Therefore, many studies have been performed to understand the behavior of the quantum entanglement in spin systems. An important interaction between spins in quantum systems is all the various kinds of Heisenberg models such as XX, XY, XZ, XXZ, etc [9,10,11,12,13]. The Heisenberg model describes the interaction of qubits not only in solid state systems, but also in quantum dots, optical lattice, and nuclear spin [14,15,16]. In recent years, the thermal entanglement of the 2-qubit systems with spin-spin interaction and spin-orbit interaction has been studied in the literature [17,18,19]. Vidal et al introduced the negativity as a measure of bipartite entanglement [20].

In the Heisenberg model, the order parameter is a vector that rotates in the three-dimensional space and

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possesses symmetry in the absence of the magnetic field. In the XY Heisenberg model, spins are rotated in plane, so the order parameter has two components. This model has been found in super-fluids such as ^{4}He. Basically, the Heisenberg model- whether isotropic or anisotropic- is often used to study the critical point and phase transition in magnet systems. In fact, the Heisenberg interaction is a more realistic model to study magnet systems. Effect of anisotropy in the exchange interaction between neighbor spins is significant, since according to many studies, the anisotropic property of the Heisenberg interaction can preserve and even increase the entanglement in the quantum spin systems. Motivated by these, in the present study, we compare the thermal entanglement behavior of the 2-qubit systems in isotropic and anisotropic Heisenberg models. We study the XX, XY, and XZ Heisenberg model with added DM interaction under the external magnetic field. The paper is organized as follows: in section II, we briefly give the Hamiltonian of the model to define the negativity. In section III, we investigate and discuss the results. Finally, section IV contains the conclusion of the research.

2 Method and results

The general Hamiltonian of the two-spin Heisenberg model with antisymmetric DM interaction and in the presence of the magnetic field is written as:

$$H = J_x S_1^x S_2^x + J_y S_1^y S_2^y + J_z S_1^z S_2^z + D\left(S_1^x S_2^y - S_1^y S_2^x\right) + h(S_1^z + S_2^z), \quad (1)$$

where $J_x.J_y.J_z$ are the exchange coefficients in three directions of x, y, z. D is the Dzisloshinski-Moriya interaction parameter along the z direction, S is the spin operator, and h is the external magnetic field. We limit our study to a two- dimensional Heisenberg interaction. To study the effect of the Heisenberg interaction on the thermal entanglement of a 2-qubit spin system, we consider three different kinds of the XX, XY, and XZ Heisenberg interaction. First, we investigate the XX Heisenberg model.

2.1 XX-DM-hz

The 2- qubit isotropic Heisenberg model subjected to the magnetic field and DM interaction is modeled by the Hamiltonian:

$$H = J(S_1^x S_2^x + S_1^y S_2^y) + D\left(S_1^x S_2^y - S_1^y S_2^x\right) + h(S_1^z + S_2^z). \quad (2)$$

Here, $J_x = J_y = J$. To determine the energy levels of the model, we should express the matrix of the Hamiltonian in the standard basis of |↓↓>, |↓↑>, |↑↓>, |↑↑>:

$$H = \begin{bmatrix} 2h & 0 & 0 & J \\ 0 & 0 & J+2iD & 0 \\ 0 & J-2iD & 0 & 0 \\ J & 0 & 0 & -2h \end{bmatrix}. \quad (3)$$

Then, we diagnosis the Hamiltonian matrix and calculate the eigen-values of the matrix. The eigen - values of this Hamiltonian are:

$$E1 = -E2 = \frac{1}{2}\sqrt{J^2 + D^2},$$

$$E3 - E4 = -\frac{h}{2}. \quad (4)$$

To calculate the quantum entanglement, we use the negativity measure. The negativity can be obtained using the density operator $\rho(T) = \exp(-\beta H)/Z$ where $Z = Tr(\exp(-\beta H))$ is the partition function and $\beta = 1/kT$. For simplicity, we take k=1. By selecting the appropriate set of the orthonormal product basis states for the density operator, the partial transpose is defined by its matrix elements

$$\rho_{m\mu.nv}^{T_1} =< \upsilon_m \upsilon_\mu |\rho| \upsilon_n \upsilon_\upsilon >= \rho_{mv,n\mu}. \quad (5)$$

The negativity of a state ρ is by definition as

$$N = \sum_i |\mu_i|, \quad (6)$$

where μ_i is the negative eigenvalue of the partial transpose density matrix ρ^{T_1}. Thus, the negativity can be written as:

$$N = \frac{\|\rho^T\| - 1}{2}. \quad (7)$$

The temperature dependence of the system is shown in Fig. 1. As seen in this figure, raising the temperature reduces the negativity because by increasing the temperature, the excited states involve the state of the system and then the state of the system converts into a mixed state that decreases the negativity. According to the figure, there is a critical temperature in which the negativity reaches zero. It is important to postpone the critical temperature. This results in protecting the entanglement of the system against the gradual or sudden death.

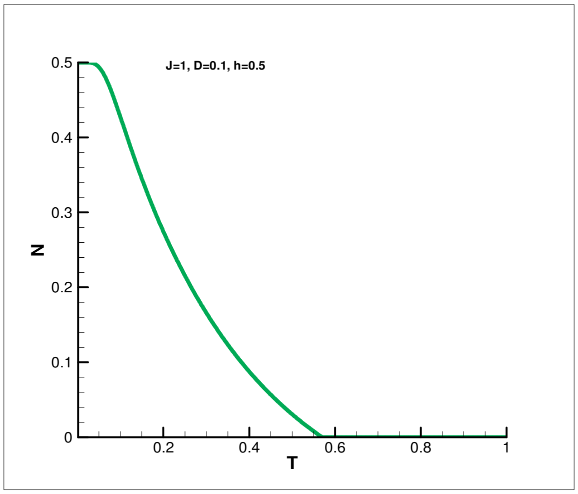

Figure 1. Temperature dependence of the XX-DM-h model.

The DM interaction can help the entanglement to maintain non- zero until higher temperatures. Figure 2 shows that raising the DM interaction increases the critical temperature and then is a positive factor to avoid the system from the disentanglement in a finite temperature. One can conclude that in contrast to the destructive role of temperature, DM interaction has an effective and reinforcing role for preserving the entanglement. Although, in very low temperatures, DM interaction cannot influence the thermal entanglement. It seems in temperatures near zero, the entanglement is independent of the DM interaction.

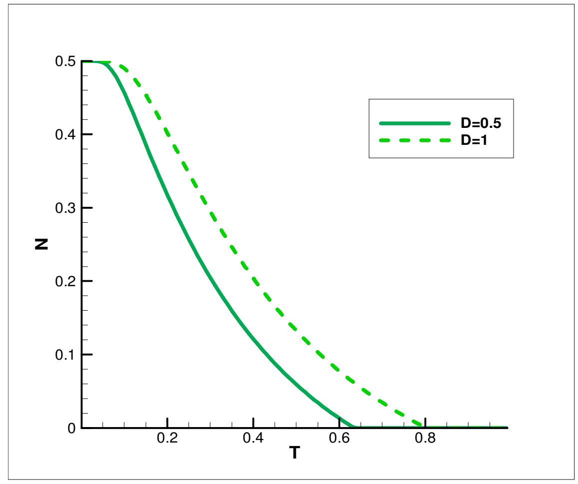

Figure 2. Thermal entanglement of the system in two different DM interactions. J=1, h=0.5.

Figure 3 shows the effect of the magnetic field on the negativity. As seen in this figure, there is a critical magnetic field above which the entanglement of the system disappears. Our study shows that this critical point depends on the exchange interaction parameter (J). In fact, a competition between the exchange interaction and the magnetic field leads to a critical point. In magnetic field values lower than J, the Heisenberg interaction entangles the system, but by increasing the magnetic field, the destructive effect of h disentangles the system.

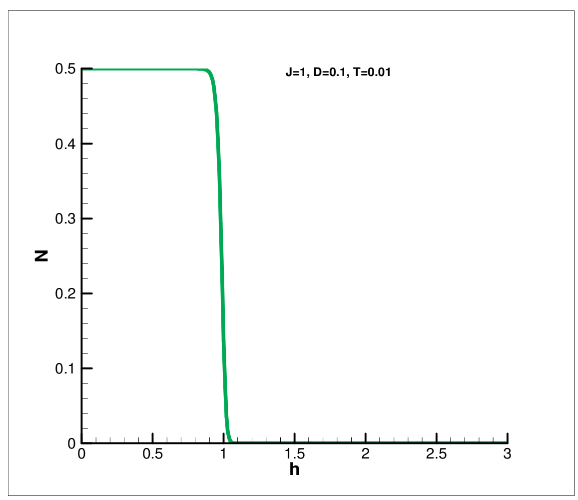

Figure 3. Magnetic field dependence of the negativity in the XX-DM-h model.

2.2. XY-DM-h

The second model that we study is the anisotropic Heisenberg model with added DM interaction and under the external magnetic field. The Hamiltonian is:

$$H = J_x S_1^x S_2^x + J_y S_1^y S_2^y + D\left(S_1^x S_2^y - S_1^y S_2^x\right) + h\left(S_1^z + S_2^z\right). \quad (8)$$

The Hamiltonian matrix is expressed as:

$$H = \begin{bmatrix} h & 0 & 0 & \frac{J_x - J_y}{4} \\ 0 & 0 & \frac{J_x + J_y + 2iD}{4} & 0 \\ 0 & \frac{J_x + J_y - 2iD}{4} & 0 & 0 \\ \frac{J_x - J_y}{4} & 0 & 0 & -h \end{bmatrix}, \quad (9)$$

The eigen -values of this matrix are:

$$E1 = -E2 = \frac{1}{4}\sqrt{(J_x + J_y)^2 + 4D^2},$$

$$E3 = -E4 = \frac{1}{4}\sqrt{(J_x - J_y)^2 + 16h^2}, \quad (10)$$

Figure 4 shows the thermal entanglement of this model. Similar to the previous model, raising the temperature reduces the negativity of the system wher in high temperatures the entanglement of the system disappears. There is a difference between the thermal behavior of the entanglement in the XX Heisenberg and XY Heisenberg models. The critical temperature in the XY model is higher than the XX model. It seems that the anisotropy parameter leads to a higher critical temperature. Thus, the anisotropy of the Heisenberg interaction can preserve entanglement against the destructive role of the temperature.

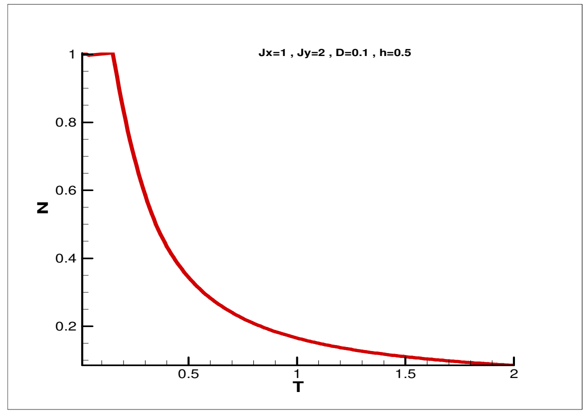

Figure 4- temperature dependence of the XY-DM-h model.

Figure 5 shows the magnetic field dependence of the negativity in the XY Heisenberg model. As seen in the figure, there is a critical magnetic field in which the entanglement of the system becomes zero. Although, below the h_c value, the entanglement of the system is independent of the magnetic field, above the h_c value, the entanglement of the system shows a descending behavior in terms of magnetic field. We can conclude that compared to the isotropic Heisenberg model, the anisotropy in the exchange interaction compensates the destructive effect of the magnetic field.

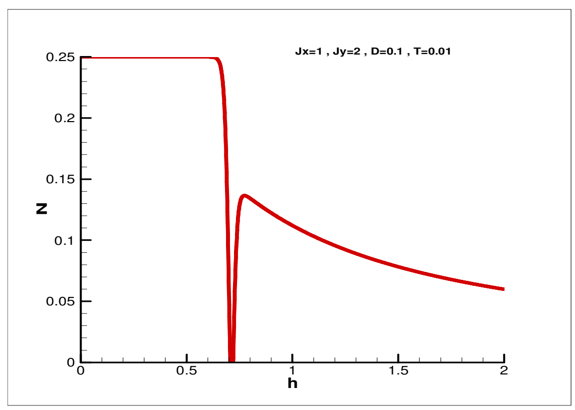

Figure 5. magnetic field dependence of the negativity in the XY-DM-h model.

2.3. XZ-DM-h

The second model that we study is the anisotropic Heisenberg model with added DM interaction under the external magnetic field. The Hamiltonian is:

$$H = J_x S_1^x S_2^x + J_y S_1^z S_2^z + D\left(S_1^x S_2^y - S_1^y S_2^x\right) + h\left(S_1^z + S_2^z\right), \qquad (11)$$

and the Hamiltonian matrix can be written as:

$$H = \begin{bmatrix} J_z + 2h & 0 & 0 & J_x \\ 0 & -J_z & J_x + 2iD & 0 \\ 0 & J_x - 2iD & -J_z & 0 \\ J_x & 0 & 0 & J_z - 2h \end{bmatrix}. \qquad (12)$$

The eigen -values of this Hamiltonian are:

$$E1 = \frac{-1}{4}(J_z + X)$$

$$E2 = \frac{-1}{4}(J_z - X)$$

$$E3 = \frac{-1}{4}(J_z + Y)$$

$$E4 = \frac{-1}{4}(J_z - Y), \qquad (13)$$

where

$$X = \sqrt{{J_x}^2 + 4D^2} \quad , \qquad Y = \sqrt{{J_x}^2 + 16h^2}\ .$$

Figure 6 shows the thermal entanglement of this model. Similar to the previous models, at high temperatures, the entanglement of the system in the XZ-DM-h model disappears. But, the critical temperature of this model is different from the two previous models.

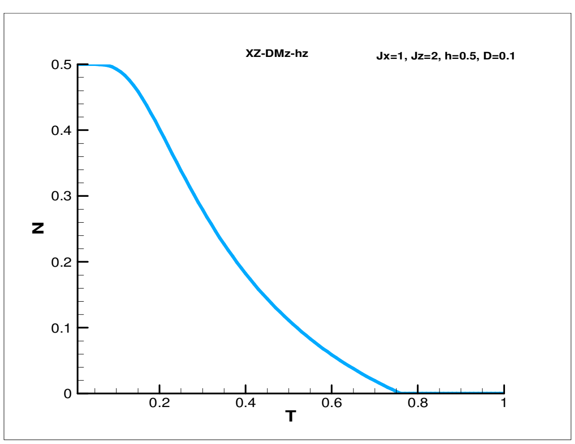

Figure 6. Temperature dependence of the negativity in the XZ-DM-h model.

Figure 7 shows the magnetic field dependence of the entanglement in the XZ-DM-h model. The behavior of the negativity in this model is similar to the previous model, but the critical magnetic field is greater in this model than the XY-DM-h model. It seems that the direction of the anisotropy affects the value of the critical magnetic field. In other words, the anisotropy in the h direction can delay the critical point.

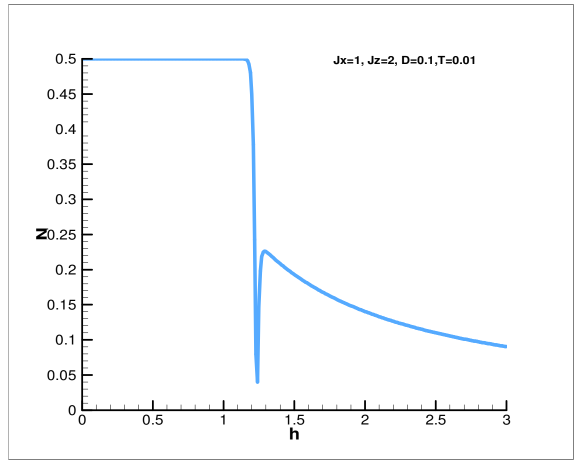

Figure 7. Magnetic field dependence of the negativity in the XZ-DM-h model.

4 Conclusions

In this study, we investigated the effect of different Heisenberg interactions on the negativity of the two- spin system. Three types of the exchange interaction, XX, XY, and XZ are considered. The thermal entanglement of these systems is investigated under the change of the external magnetic field. We found that the temperature behavior of the system is similar, although the critical temperature is different. Also, we investigated the effect of the external magnetic field on the negativity of the system. We observed that all three models have a critical magnetic field in which the negativity of the system becomes zero. In fact, the anisotropic Heisenberg interaction can affect the critical temperature and critical magnetic field of the two- spin systems.

References

[1] E. Schrodinger, "Mathematical proceedings of the Cambridge philosophical society." Mathematical Proceedings of the Cambridge Philosophical Society, **31** (1935) 555.

[2] Z. Xi, Y. Li, and H. Fan, “Quantum coherence and correlations in quantum system.” Scientific Reports, **5** (2015) 10922.

[3] L. Amico, R. Fazio, A. Osterloh, and V. Vedral, “Entanglement in many-body systems.” Reviews of Modern Physics, **80** (2008) 517.

[4] G. J. Mooney, C. D. Hill, and L.C. L. Hollenberg, “Entanglement in a 20-Qubit Superconducting Quantum Computer.” Scientific Reports, **9** (2019) 13465.

[5] S. P. Kulik, “Quantum teleportation with qubits”, Kulik, S. P. "Quantum teleportation with qubits." Russian Microelectronics, **35** (2006).

[6] W. K. Wootters, “Quantum Entanglement as a Resource for Communication. In: Quantum Mechanics at the Crossroads.” The Frontiers Collection. Springer, Berlin, Heidelberg (2007).

[7] S. Simmons, R. Brown, H. Riemann, et al.” Entanglement in a solid-state spin ensemble.” Nature, **470** (2011) 69.

[8] G. Burkard, “Spin-entangled electrons in solid-state systems.” Journal of Physics: Condensed Matter, **19** (2007) 233202.

[9] G. L. Kamta, A. F. Starace, “Anisotropy and Magnetic Field Effects on the Entanglement of a Two Qubit Heisenberg XY Chain.” Physical Review Letters, **88** (2002) 107901.

[10] K. Guo, M. Liang, H. Xu, et al. “Entanglement in a two-spin (1/2, 3/2) mixed-spin Heisenberg XX chain with an inhomogeneous external magnetic field.” Journal of Physics A: Mathematical and Theoretical, **54** (2011) 491.

[11] L. S. Lima, “Thermal Entanglement in the Quantum XXZ Model in Triangular and Bilayer Honeycomb Lattices.” Journal of Low Temperature Physics, **198** (2020) 241.

[12] T. Martin, T. A. Giresse, “Entanglement Dynamics of a Two-qubit XYZ Spin Chain Under both Dzyaloshinskii-Moriya Interaction and Time-dependent Anisotropic Magnetic Field.” International Journal of Theoretical Physics, **59** (2020) 2232.

[13] Z. Ling, Y. Xue-Xi, S. He-Shan, G. Yan-Qing, “Thermal entanglement in 1D optical lattice chains with nonlinear coupling.” Chinese Physics, **14** (2005) 1168.

[14] V. Murg, F. Verstraete, J. I. Cirac, “Variational study of hard-core bosons in a two-dimensional optical lattice using projected entangled pair states.” Physical Review A, **75** (2007) 033605.

[15] G. Burkard D. Loss, and D. P. DiVincenzo, “Coupled quantum dots as quantum gates.” Physical Review B, **59** (1990) 2070.

[16] W. D¨ur, G. Vidal, and J. I. Cirac Dür, Wolfgang, Guifre Vidal, and J. Ignacio Cirac. "Three qubits can be entangled in two inequivalent ways." Physical Review A, **62** (2000) 062314.

[17] F. Kheirandish, S. J. Akhtarshenas, and H. Mohammadi, “Effect of spin-orbit interaction on entanglement of two-qubit Heisenberg X Y Z systems in an inhomogeneous magnetic field.” Physical Review A, **77** (2008) 042309.

[18] F. Rojas, “Thermal entanglement of a coupled electronic spins system: interplay between an external magnetic field, nuclear field and spin–orbit interaction.” Quantum Information Processing, **14** (2015) 1973.

[19] O. S. Duarte, C. S. Castro, D. O. Soares-Pinto, M. S. Reis, “Witnessing spin-orbit thermal entanglement in rare-earth ions.” Europhysics Letters, **103** (2013) 40002.

[20] G. Vidal and R. F. Werner, “Computable measure of entanglement.” Physical Review A, **65** (2002) 032314.