Journal of Interfaces, Thin films, and Low dimensional systems 4 (1) Summer and Autumn (2020) 323-329

Dynamics of the entanglement in a two-spin system with long-range interaction

Scientific research paper

Mohammad Reza Soltani[1*], Maryam Mahmoudi[1], Saeed Mahdavifar[2], and Arezu Jahanshir[3]

[1]*Department of Physics,Yadegar-e-Imam Khomeini (RAH) Shahre Rey Branch, Islamic Azad University, Tehran, Iran*

[2]*Department of Physics, University of Guilan, 41335-1914, Rasht, Iran*

[3]*Department of Physics and Engineering Sciences, Imam Khomeini International University, Buein Zahra Higher Education Centre of Engineering and Technology, Iran*

ARTICLE INFO

Article history:
Received 25 May 2021
Revised 24 July 2021
Accepted 9 August 2021
Available online 28 October 2021

Keywords:
Entanglement dynamics
Long range
Negativity
Dzyaloshinskii-Moriya interaction
Magnetic field

ABSTRACT

Introduction: In this study, we investigate the dynamics of the Entanglement in a two- spin system with long- range interaction.

Method: For this purpose, we use the negativity as the entanglement measurement. Using the time evolution operator, we obtain entanglement dynamics of the system at time t. We consider two different initial states and investigate the dynamical behavior of the system for all of them separately. Finally, we consider both J and D as a function of R and then study the time evolution of the entanglement in different R.

Results: We find that in the long range interaction, the R dependence of the dynamical behavior in the two systems is different. Depending on the initial states, the DM interaction and the magnetic field have no effect on the entanglement dynamics.

1 Introduction

One of the most important predictions of modern quantum physics is the quantum entanglement [1]. Quantum entanglement is a quantum phenomenon in which the quantum states of two or more objects have to be described with reference to each other, even though the individual objects may be spatially separated. This results in quantum correlations between the observed physical properties of objects. Much effort is devoted describing the nature of the entanglement [2]. Quantum entanglement plays an important role in quantum information processing [3], teleportation [4], communication systems [5], quantum computer [6,7], quantum spin networks [8], security cryptography [9]. Therefore, significant research has been performed to understand quantum entanglement behavior in spin systems, such as all various kinds of Heisenberg (XX, XY, XXZ and XYZ) models, similar Ising models [10-12], and systems with Dzisloshinski-Moriya interaction [13,14].

Real physical systems are never isolated, as the connection between the system and the environment is inevitable. The quantum dynamics of physical systems is always complicated by their coupling with a number of «environmental» modes. Then, for realistic quantum systems, the loss of coherence is inevitable due to the interaction with the environment [15]. Recently, there has been more and more research on the dynamics of quantum entanglement under environmental influence [16-20].

*Corresponding author.
Email address: m.r.soltani.em@gmail.com
DOI: 10.22051/jitl.2021.36221.1055

In this paper, Entanglement is thus taken as a dynamic quantity on its own, as we survey how it evolves due to the unavoidable interaction of the entangled system with its surroundings. We analyze several scenarios for various initial states. In addition, we study the impact of spin distance on the dynamics of entanglement. Actually, we look at the exchange interaction as a function of the distance between two spins. This kind of interaction is known as long-term interaction. In recent years, long-term interactions have attracted a great deal of attention because they may produce new interesting phenomena [21-23]. In addition, some research has focused on the experimental application of spin systems that interact over long distances. Indeed, the inverse-square, trigonometric and hyperbolic interacting particle systems [25,26] and their spin generalizations [27-30] are important models of many-body systems due to their exact solvability and intimate connection to spin systems in condensed matter [31,32] and in other areas in physics. Therefore, studying the dynamical behaviour of systems with such interaction could be important. A major aspect of our analysis is that the entanglement does not disappear over time.

The paper is organized as follows: in the next section, we introduce the model Hamiltonian and describe briefly the techniques used to obtain the results discussed in the subsequent sections. In section III, we present and discuss the numerical results of modeling. Finally, Section 4 contains the concluding remarks.

2 Model

Here, we consider a set of two localized spin- 1/2 particles coupled through exchange interactions J, subjected to an external magnetic field of strength h with Dzyaloshinskii-Moriya interaction:

$$H = J(R)S_1^x S_2^x + D\left(S_1^x S_2^y - S_1^y S_2^x\right) + h(S_1^z + S_2^z), \quad (1)$$

Where J(R) is the exchange interaction parameter that varies with the distance between spins, R, as $J(R) = 1/R^2$. D is the Dzisloshinski-Moriya interaction parameter, and S is the spin-1/2 operator. In the base ket of S_{tot}^z , the Hamiltonian matrix is formed as

$$\mathrm{H} = \begin{bmatrix} h & 0 & 0 & \frac{J}{4} \\ 0 & 0 & \frac{J}{4}+\frac{iD}{2} & 0 \\ 0 & \frac{J}{4}-\frac{iD}{2} & 0 & 0 \\ \frac{J}{4} & 0 & 0 & -h \end{bmatrix}. \quad (2)$$

The eigenvalues and eigenstates are:

$$\begin{aligned} \varepsilon_1 &= \frac{x}{4}\ ,\ |1\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + \alpha|\downarrow\uparrow\rangle) \\ \varepsilon_2 &= -\frac{x}{4}\ ,\ |2\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - \alpha|\downarrow\uparrow\rangle) \\ \varepsilon_3 &= \frac{y}{4}\ ,\ |3\rangle = \frac{1}{\sqrt{1+\beta^2}}(\beta|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \\ \varepsilon_4 &= -\frac{y}{4}\ ,\ |4\rangle = \frac{1}{\sqrt{1+\beta'^2}}(\beta'|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle) \end{aligned} \quad , (3)$$

$$x = \sqrt{J^2 + 4D^2}, y = \sqrt{J^2 + 16h^2}, \alpha = \frac{J - 2iD}{x},$$

$$\beta = \frac{4h - y}{J}, \beta' = \frac{4h + y}{j}, \quad (4)$$

On the other hand, to study the entanglement, one should obtain the density matrix of the system:

$$\rho = |\Psi_0 >< \Psi_0|\ , \quad (5)$$

where $|\Psi_0 >$is the initial state of the system Hamiltonian. In our system the density matrix is reduced to:

$$\rho = \begin{bmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{bmatrix}. \quad (6)$$

Using the eigenvalues of the partial transpose of the density matrix, one can obtain the negativity of this model. Negativity is given by:

$$N = \frac{\|\rho^T\| - 1}{2}, \tag{7}$$

where $\|\rho^T\|$ is the trace norm or the sum of the absolute values of the operator ρ^T. In following, we try to describe dynamics of the entanglement of this system. The dynamic evolution operator $U(t) = \exp(-iHt)$ can be obtained as [33]

$$U = \begin{bmatrix} U_{11} & 0 & 0 & U_{14} \\ 0 & U_{22} & U_{23} & 0 \\ 0 & U_{32} & U_{33} & 0 \\ U_{41} & 0 & 0 & U_{44} \end{bmatrix}, \tag{8}$$

where

$$U_{11} = U_{44} = cos\frac{yt}{4} + \frac{4ih}{y} sin\frac{yt}{4},$$

$$U_{14} = U_{41} = \frac{ij}{y} sin\frac{yt}{4},$$

$$U_{22} = U_{33} = cos\frac{xt}{4}, \qquad U_{23} = -i\alpha sin\frac{xt}{4},$$

$$U_{32} = -i\alpha^* sin\frac{xt}{4}.$$

In the next section, we consider two different initial configurations and study the dynamics of the system for both configurations. We compare the results of each configuration.

3 Discussion

Now, we consider two different initial states of the system and survey the time evolution of entanglement of the system for different parameters such as J, D, and h. Also, we consider J and D as a function of distance between two spins, R.

Case 1:

Suppose that at time t = 0 the qubits are entangled together and initial state given by:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle). \tag{9}$$

Now, by applying the evolution operator on the $|\Psi_0>$, we can obtainthe physical state of the system at time t as:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\left(\cos\left(\frac{xt}{4}\right) - i\alpha\sin\left(\frac{xt}{4}\right)\right)|\uparrow\downarrow\rangle + \left(\cos\left(\frac{xt}{4}\right) - i\alpha^*\sin\left(\frac{xt}{4}\right)|\downarrow\uparrow\rangle\right)\right]. \tag{10}$$

It is noticeable that by selecting the above initial state, the role of the magnetic field disappears in the time evolution of the entanglement. In fact, the change of h does not affect the entanglement dynamics of the system. By having $|\psi(t)\rangle$, we can calculate the density matrix of the system at time t, and then get a formula for the negativity as a function of t:

$$N = \frac{\sqrt{1 - \frac{4D^2}{x^2}\sin^2\frac{xt}{2}}}{2}. \tag{11}$$

Figure 1 shows the evolution of the Negativity when J and D are constant. As seen in the figure, the system has a regular behavior over time. Although the amount of the negativity drops highly at some times, but it will never be zero.

Then, we investigate the effect of the DM interactions on the dynamics of the negativity. Figure 2 shows the D dependence of the negativity evolution. Increasing the DM interaction, increases the amplitude of the entanglement oscillations. Indeed, the weaker DM interaction leads to more stable oscillations of the entanglement over time. In addition, the rise of D, reduces the time period of negativity.

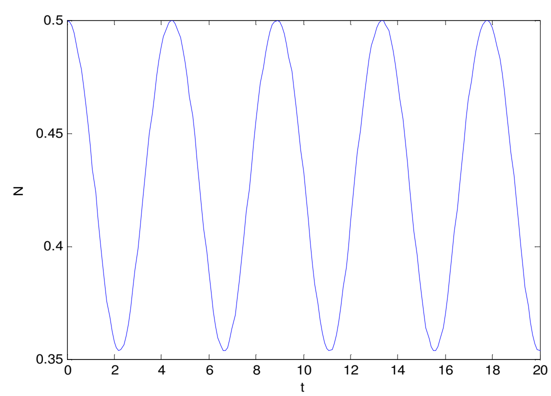

Figure 1. The time evolution of the negativity's in the system with initial state as $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. We set J=1, D=0.5.

When we study the time period of these oscillations, we observe that for J>D the time period diminishes by increasing D. But, for J≤ D, the time period remains constant. In fact, it may be said that J=D is a point where the behavior of the entanglement dynamics has changed.

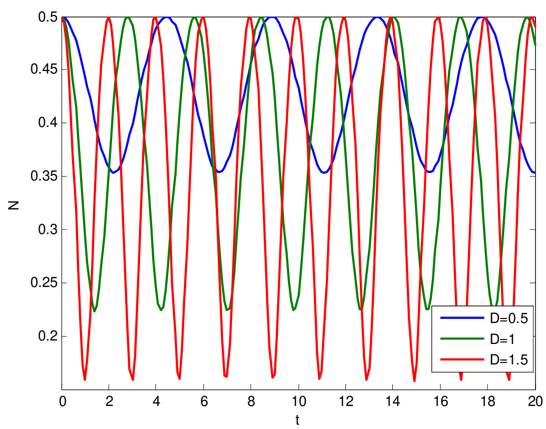

Figure 2. The D dependence of the negativity's time evolution with initial state as $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. Set J=1.

Now, we investigate the dynamics of the negativity when the exchange interaction is a function of the spin distance, R as $J = \frac{1}{R^2}$. The effect of the long range exchange interaction on the negativity dynamics is noticeable. As seen in Fig. 3, when the distance between two interacting spins increases, the time evolution of the entanglement becomes more unstable and the amplitude and time period of the oscillations increase. It shows the entanglement dynamics depends upon the distance between qubits. The rapid oscillations show rapid energy exchange between spins at very small distances. Indeed, reducing R is a positive factor to avoid the sharp and unstable oscillations of the entanglement dynamics. If we study the time period of these oscillations, we observe that for J>D and then R< 1/D, the time period decreases by increasing D. But, for J≤ D and then R≥1/D, the time period remains constant.

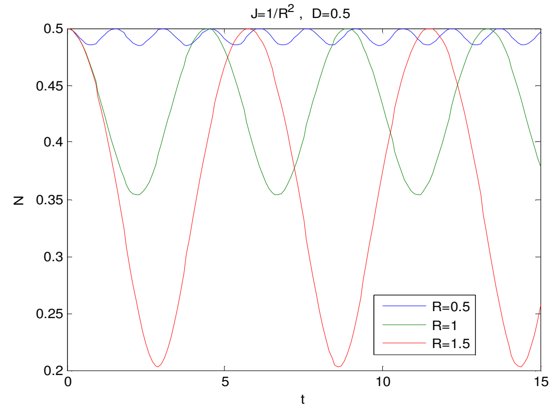

Figure 3. The R dependence of the entanglement's time evolution with initial state as $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. Set $J = \frac{1}{R^2}$.

In the next step, we consider both J and D as functions of R. It is noticeable that in different values of R, the amplitude of the fluctuations of the entanglement is mostly the same. But, by increasing R, the time period increases, see Fig. 4.

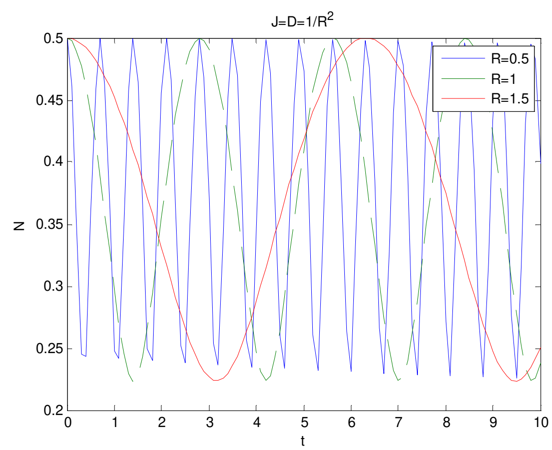

Figure 4. The R dependence of the entanglement's time evolution with initial state as $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. Set $J = \frac{1}{R^2}$ and $D = \frac{1}{R^2}$.

Case 2:

Suppose that at time t = 0 the qubits are entangled together and initial state given by:

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle). \quad (12)$$

Now, by applying the evolution operator on $|\Psi_0>$, we can obtain the physical state of the system at time t. This initial state makes the effect of the DM interaction remove from the dynamic behavior of the entanglement. The negativity of the system with this initial state is:

$$N=\frac{1}{2}\sqrt{(1-\frac{32h^2}{y^2}\sin^2\frac{yt}{4})^2+\frac{16h^2}{y^2}\sin^2\frac{yt}{2}}\ . \quad (13)$$

Like the previous section, we study the entanglement dynamics of this initial state in three modes: a) J and D are constant, b) $=\frac{1}{R^2}$, and c) $J=D=\frac{1}{R^2}$. First, we consider $J=1$ and $D=1$. As seen in Fig. 5, the dynamic behavior of this kind of initial state is oscillatory. Compared to the previous initial state, the time period and amplitude drop of the negativity are lower.

When the exchange interaction is dependent on R, the dynamic behavior of the negativity is as illustrated in Fig. 6. By increasing R, and therefore reducing J, the oscillations amplitude increases. Indeed, the less distance between spins leads to a stronger exchange interaction that leads to a more stable behavior of the entanglement dynamics along the time. Compared to the previous initial state, the amplitude drop of the dynamic fluctuations, especially in large R, is more. Similar to the previous initial state, when we study the time period of these oscillations, we observe that for J>h the time period decreases by increasing D. But, for J≤ h, the time period remains constant. In fact, it can be said that J=h is a point at which the behavior of entanglement dynamics has changed.

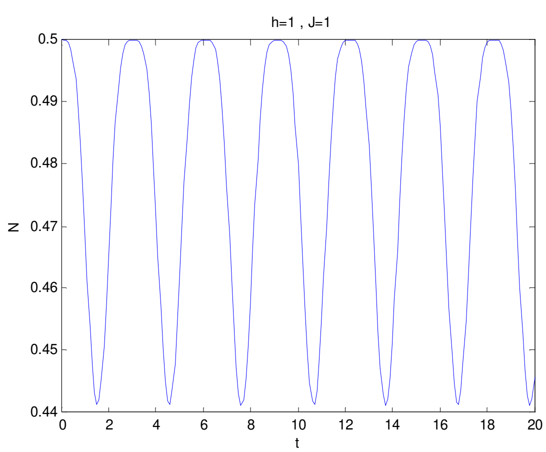

Figure 5. The time evolution of the entanglement in the system with initial state as $|\psi(0)\rangle=\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle)$. We set J=1, h=1.

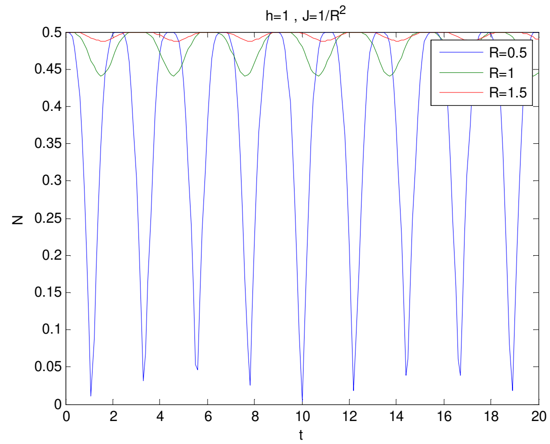

Figure 6. The R dependence of the entanglement's time evolution with initial state as $|\psi(0)\rangle=\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle)$. Set $J=\frac{1}{R^2}$.

As mentioned previously, in the system with such an initial state, the DM interaction does not influence the dynamic behavior of the negativity. In contrary, the magnetic field influences the dynamics of the negativity in the system with this new initial state. Figure 7 shows that in a weak magnetic field, the oscillations of negativity are greater than when exposed to a strong magnetic field. It can be said that the magnetic field has a positive effect on the dynamics of the entanglement in this system. Similar to the previous section, until R has satisfied the condition of J≤h ($J=1/R^2$), the time period of the negativity oscillations is constant. As soon as J>R, the time period reduces by decreasing R (increasing J).

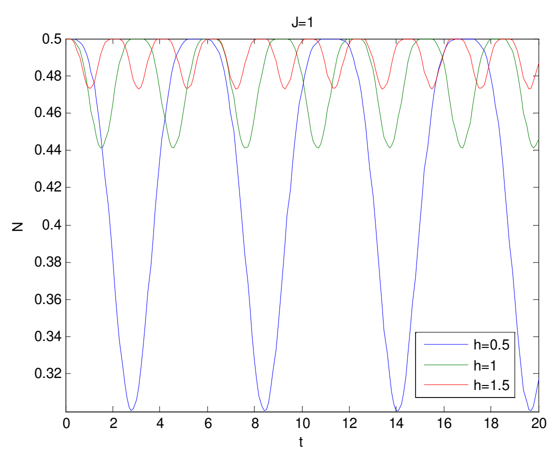

Figure 7. The time evolution of the entanglement in different magnetic fields in the system with initial state as $|\psi(0)\rangle=\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle+|\downarrow\downarrow\rangle)$.

4 Conclusions

We considered two different initial states of the system and studied the dynamic behavior of the system for each of both situations, separately. It was interesting that in the system with initial state of $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$, the magnetc field didn't have any effect on the entanglement dynamics while the DM interaction had the negative effect on the time evolution of the entanglement. In contrast, in the dynamic behavior of the system with initial state of $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$, the DM interaction has no effect on the entanglement dynamics, while the magnetic field was a positive factor to decrease the amplitude of the oscillations of the entanglement with time. In addition, we found that in the long range interaction, namely the exchange interaction was a function of the distance between two spins, the R dependence of the dynamic behavior in the two systems was different. In the system with $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ as the initial state, increasing the spin distance caused the sharp drop of the oscillation amplitude of the negativity with time. Indeed, we can say that the high R, namely the weak exchange interaction, causes the unstable fluctuations in the entanglement dynamics of the system. Whereas, in the system with $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ as the initial state, increasing the spin distance led to a more stable dynamic behavior in the system. Furthermore, when both J and D were functions of R, the change of R did not have any effect on the oscillation amplitude of the entanglement dynamics but increased the time period of the oscillations.

References

[1] W. Wotters, "Entanglement of Formation of an Arbitrary State of Two Qubits." Physical Review Letters, **80** (1998) 2245.

[2] R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, "Quantum Entanglement." Review of Modern Physics, **81** (2009) 865.

[3] J. Gruska, “Quantum entanglement as a new information processing resource.” New Generation Computing, **21** (2003) 279.

[4] S. Mirzaei, A. R. Akbarieh, “Quantum Teleportation via Entangled State of Light in Schwarzschild Black Hole.” International J Theoretical Physics, **59** (2020) 3583.

[5] L. Gyongyosi, S. Imre and H. V. Nguyen, "A Survey on Quantum Channel Capacities." in IEEE Communications Surveys & Tutorials, **20** (2018) 1149.

[6] X.-D. Cai, D. et al. “Entanglement-Based Machine Learning on a Quantum Computer” Physical Review Letters, **114** (2015) 110504.

[7] L. F. Santos, "Quantum computation with quantum dots." Physical Review A, **67** (2003) 062306.

[8] Peter J. Pemberton-Ross and Alastair Kay, “Perfect Quantum Routing in Regular Spin Networks.” Physical Review Letters, **106** (2011) 020503.

[9] Yin, J., Li, YH., Liao, SK. et al. “Entanglement-based secure quantum cryptography over 1,120 kilometres.” Nature, **582** (2020) 501.

[10] X. Wang, "Entanglement in the quantum Heisenberg XY model", Physical Review A, **64** (2001) 012313.

[11] S. Deniz, E. Aydiner, " Thermal entanglement in the mixed three-spin XXZ Heisenberg model on a triangular cell." Chinese Physics B, **2** (2014) 5.

[12] L. F. Santos, "Entanglement in quantum computers described by the XXZ model with defects." Physical Review A, **67** (2003) 062306.

[13] S. D. Han, E.Aydiner, " Thermal entanglement in the mixed three-spin XXZ Heisenberg model on a triangular cell." Chinese Physics B, **23** (2014) 050305.

[14] Z. Ming , X. Hui, L. Xiao-Xian, T. Qing, "The effects of the Dzyaloshinskii—Moriya interaction on the ground-state properties of the XY chain in a transverse field." Chinese Physics B, **22** (2013) 9.

[15] H. Breuer, F. Petruccione, "The Theory of Open Quantum-Systems", Oxford University Press, Oxford, New York (2002).

[16] T. Werlang, S. Souza, F. F. Fanchini , C. J. Villas Boas, " Robustness of quantum discord to sudden death." Physical Review A, **80** (2009) 024103.

[17] L. Mazzola, J. Piilo , S. Maniscalco, " Sudden Transition between Classical and Quantum Decoherence." Physical Review Letters, **104** (2010) 200401

[18] R. Vasile, P. Giorda, S. Olivares, G. A. Paris , S. Maniscalco, " Open Quantum Systems and entanglement." Physical Review A, **82** (2010) 012313.

[19] B. Wang, Z. Xu, Z. Chen , M. Feng, " Bell violation versus geometric measure of quantum discord and their dynamical behavior." Physical Review A, **81** (2010) 014101.

[20] F. Fanchini, T. Werlang, C. A. Brasil, L. G. E. Arruda , A. O. Caldeira, " Non-Markovian dynamics of quantum discord." Physical Review A, **81** (2010) 052107.

[21] M. R. Soltani, S. Mahdavifar, M. Mahmoudi, " Entanglement in a two-spin system with long-range interactions." Chinese Physics B, **25** (2016) 087500.

[22] P. Jurcevic, " Quasiparticle engineering and entanglement propagation in a quantum many-body system." Nature, **511** (2014) 202.

[23] J. Schachenmayer, B. P. Lanyon, C. F. Roos, A. Daley, "Entanglement Growth in Quench Dynamics with Variable Range Interactions." Physical Review X, **3** (2013) 031015.

[24] Samihr Hermes, Tony J. G. Apollaro, Simone Paganelli, Tommaso Macri, “Dimensionality-enhanced quantum state transfer in long-range-interacting spin systems.” Physical Review A, **101** (2020) 053607.

[25] Soltani M R, Mahdavifar S, Mahmoudi M. “Entanglement in a two-spin system with long-range interactions.” Chinese Physics B, **25** (2016) 087501.

[26] N. Zidan,” Entropic Uncertainty in Spin XY Model with Long-Range Interactions.” Entropy, **22** (2020) 837.

[27] J. Gibbons, T. Hermsen, " A generalisation of the Calogero-Moser system." Physica D, **11** (1984) 337.

[28] S. Wojciechowski, " Integrable one-particle potentials related to the Neumann system and the Jacobi problem of geodesic motion on an ellipsoid." Physics Letters A, **111** (1985) 107.

[29] F. D. Haldane, "Exact Jastrow-Gutzwiller resonating-valence-bond ground state of the spin-(1/2 antiferromagnetic Heisenberg chain with 1/r 2 exchange." Physical Review Letters, **60** (1988) 635.

[30] B. S. Shastry, "Exact solution of an S= 1/2 Heisenberg antiferromagnetic chain with long-ranged interactions." Physical Review Letters, **60** (1988) 639.

[31] H. H. Tu and Y. Wu, “Exactly Solvable Quantum Impurity Model with Inverse-Square Interactions.” Physical Review Letters, **123** (2019) 066406.

[32] H. Fakhri, S.B. SeyedeinArdebili, “Exact solutions for the ferromagnetic and antiferromagnetic two-ring Ising chains of spin-1/2.” Physica A: Statistical Mechanics and its Applications, **523** (2019) 557.

[33] T. Quan, Z. D. Wang , C. P. Sun, " Quantum critical dynamics of a qubit coupled to an isotropic Lipkin-Meshkov-Glick bath." Physical Review A, **76** (2007) 012104.