Journal of Interfaces, Thin films, and Low dimensional systems 2 (2017-2018) 93-98

Transparency of overdense plasma with V shape density profile

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ARTICLE INFO

Article history:
Received 30 October 2017
Revised 24 February 2018
Accepted 11 March 2018
Available online 18 March 2018

Keywords:
Over dense plasma
Surface wave
Linear density
Stratified plasma slab
Anomalous transmission

ABSTRACT

In this study, we investigate the transparency of an over-dense inhomogeneous plasma slab. This anomalous transmission is achieved when the conditions are provided for the incident electromagnetic wave to excite coupled surface waves on both sides of the slab. These conditions require that the homogeneous overdense plasma, or the metallic film, is placed between two dielectric layers. Here, the inhomogeneity of the plasma allows us to naturally establish the conditions by considering a specific shape for the geometry of the density profile. Within this profile, the density linearly grows up from both sides of the slab while the corresponding dielectric permittivity simultaneously gains negative values. We obtain the exact solutions of the wave equations inside the plasma and study the conditions for high transparency. The transmission losses due to the collision effects are also discussed.

1 Introduction

In recent years, there has been a growing interest on the optical properties of the left handed materials (LHM). These materials have negative dielectric permittivity and permeability that include metallic films and dense plasma layers [1]-[4]. These structures have novel properties in many fields of science and technology, for example; supper lenses and sub-wavelength lithography [5]-[8], invisible coatings [9]-[14], Faraday rotation [15], plasma heating [16], and plasmons resonant absorption [17]-[18].

Media with negative dielectric permittivity are generally opaque to the electromagnetic waves. However, under resonant conditions, these structures become totally transparent. Many experimental and theoretical studies have been devoted to understand the interaction of electromagnetic waves with these materials. The investigations show that the anomalous transmission of the waves takes place in these materials because of the amplification of the evanescent waves by the excitation of the surface plasmons [19]. An incident ordinary electromagnetic wave cannot excite plasmons, however the evanescent electromagnetic waves can excite them. The diffracting gratings can be used to produce the evanescent waves and allow the excitation of plasmons [20]. However, applying the mechanism of total internal reflection of a near boundary inhomogeneity of the dielectric permittivity can provide the required evanescent waves [21].

The high transparency conditions of a metallic film, or equivalently an over-dense plasma layer, has been studied both theoretically and experimentally in [1]. In this work and other similar studies, for example [16] and [22], the metallic film (with $\varepsilon < 0$) is supposed

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DOI: 10.22051/JITF.2018.17746.1012

to be placed between two dielectric layers (with $\varepsilon > 0$). This structure is also equipped with two prism on both sides. The incident electromagnetic wave on this symmetric structure of prism, positive dielectric and metallic film, is reflected at the prism dielectric boundary. The electromagnetic wave then becomes evanescent within the dielectric layer and the conditions for the excitation of the surface waves on the interface between the dielectric and the metallic film are fulfilled. The surface waves excite simultaneously on both sides of the slab and transfer the energy of the incident electromagnetic wave trough the metallic film. In these studies, the dielectrics and the metallic film are considered as homogeneous mediums with constant dielectric permittivity which are changed discontinuously at the interfaces.

Here, we consider the inhomogeneous plasma layers which play effectively the role of both dielectric and the metallic films. We show that by imposing a specific spatial geometry for the density of the plasma layers, the conditions for the anomalous light transmission can be achieved. We consider a geometry of the density profile of the plasma in which the dielectric permittivity linearly and continuously decreases from a positive value to a negative value. If this reduction takes place from both sides of the slab, the surface waves simultaneously excite from both sides. Consequently, the energy of the incident electromagnetic wave is carried out from the slab.

The geometry we apply here on the density of the plasma also has been considered in our previous work [23]. The behavior of the surface waves and the distribution of the electric field inside the plasma medium have been discussed there. Here, we study the suitable conditions for the anomalous transmission and the high transparency of the slab. In this regards, we calculate the transmission and the reflection amplitudes from the entire structure and discuss the main parameters involved in the high transparency conditions of the slab.

The organization of this paper is as follows: The fundamental equations are studied in section two. The proposed density profile is modeled in section three. In section four, the anomalous transmission of the electromagnetic waves through the entire slab is obtained and discussed. Finally, section five presents the results.

2 The Model

We consider a plane monochromatic wave of frequency ω, incident to a layer of inhomogeneous cold plasma layer immersed in vacuum. In the plasma layer, the electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ and the velocity $\boldsymbol{V}$ obey the following linearized set of fluid-Maxwell equations:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{1}$$

$$\nabla \times \boldsymbol{B} = -\frac{4\pi n_0(x)e}{c^2}\boldsymbol{V} + \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}, \tag{2}$$

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{e}{m}\boldsymbol{E} - \acute{v}\boldsymbol{V}. \tag{3}$$

Here $n_0(x)$ is the plasma density which depends on the $x-$coordinate where $\acute{v}$ denotes the collision frequency. Considering the time dependency of the field equations as $e^{-i\omega t}$, from Eq. (1)-(3), the wave equation becomes:

$$\nabla(\nabla.\boldsymbol{E}) - \nabla^2\boldsymbol{E} - k_0^2\,\varepsilon\,\boldsymbol{E} = 0\,, \tag{4}$$

where $k_0 = \frac{\omega}{c}$ and ε is the effective dielectric permittivity of the plasma defined as:

$$\varepsilon = 1 - \frac{\omega_p^2}{s\,\omega^2}\,, \quad s = 1 + iv, \tag{5}$$

and $v = \frac{\acute{v}}{\omega}$. We assume that the ions are motionless and $\omega_p^2 = \frac{4\pi n_0(x)e^2}{m}$ is the plasma frequency while e and m are the charge and the mass of the electron. Letting the x-coordinate into the plasma layer and y and z running along the interface, we consider the variation of all physical quantities as $\psi(x)e^{ik_y y}$ where $k_y = k_0 \sin\,\theta$ and θ is the incident angle. In this case, for a p-polarized wave specified with $\boldsymbol{E} = (0,0,E)$, Eq. (4) yields:

$$\frac{d^2E}{dx^2} + k_0^2\left[\varepsilon(x) - \overset{2}{\sin}\,\theta\right]E = 0 \tag{6}$$

3 The dielectric permittivity profile

A metallic film or equivalently, a dense plasma layer can be considered as a negative ε material. According to Eq. (5), for adequate large densities n, one gets

$\omega < \omega_p$ and $\varepsilon < 0$. In these cases, the plasma behaves as a LHM medium and is normally opaque for the incident electromagnetic wave. However, it has been shown that the layer could become totally transparent by fulfilling the condition for the propagating the incident wave to excite coupled surface modes or plasmons, at the surface of the layer. In fact, plasmons are trapped surface modes propagating along the interfaces between two media with different permittivity sign such as a dielectric-metal boundary.

Here, we study the electromagnetic waves transmission of an inhomogeneous plasma layer. The required dielectric layers can be provided by the plasma layer itself. We suppose that the plasma gradually obtains negative dielectric permittivity. According to Fig. (1), the dielectric permittivity decreases from $\boldsymbol{\varepsilon} = \mathbf{1}$, to negative values equal to - **1**.

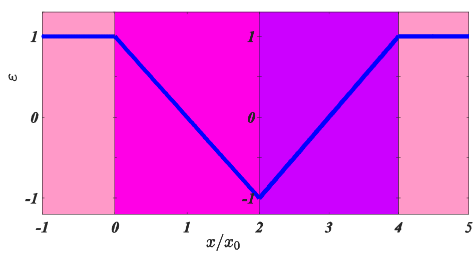

Fig. 1. The profile of the effective dielectric permittivity $\boldsymbol{\varepsilon}$ as a function of position $\frac{x}{x_0}$.

Then in order to provide the conditions for the excitation of the coupled surface modes, $\boldsymbol{\varepsilon}$ symmetrically grows up to the positive values and reaches $\boldsymbol{\varepsilon} = \mathbf{1}$ at the rear side of the slab. In this way, the layer behaves as an ordinary dielectric media with $\boldsymbol{\varepsilon} > \mathbf{0}$ for $\mathbf{0} < \boldsymbol{x} < \boldsymbol{x_0}$ and $\mathbf{3}\boldsymbol{x_0} < \boldsymbol{x} < \mathbf{4}\boldsymbol{x_0}$ where $\boldsymbol{x_0}$ is a characteristic length. However, for $\boldsymbol{x_0} < \boldsymbol{x} < \mathbf{3}\boldsymbol{x_0}$ we have an LHM media, namely $\boldsymbol{\varepsilon} < \mathbf{0}$. The dielectric profile has the following form:

$$\varepsilon(x) = \begin{cases} 1 - x/x_0\,, & 0 < x < 2x_0 \\ -3 + x/x_0\,, & 2x_0 < x < 4x_0 \end{cases}. \tag{7}$$

In region $0 < x < 2x_0$ the wave equation (Eq. (6)) becomes:

$$\frac{d^2E}{dx^2} + k_0^2\left[\overset{2}{\cos}\,\theta - \frac{x}{sx_0}\right]E = 0\,. \tag{8}$$

Put:

$$\zeta = \left(\frac{k_0^2}{sx_0}\right)^{\frac{1}{3}}(sx_0\cos^2 - x), \tag{9}$$

Eq. (8) becomes:

$$\frac{d^2E(\zeta)}{d\zeta^2} + \zeta E(\zeta) = 0\,. \tag{10}$$

Solutions of this equation is given by the Airy functions of the first kind (Ai) and the second kind (Bi) which can be written as follows:

$$E(\zeta) = C_1Ai(-\zeta) + C_2Bi(-\zeta)\,, \tag{11}$$

where C_i's are constants.

In region $2x_0 < x < 4x_0$, one finds a similar wave equation for the variable:

$$\eta = \left(\frac{k_0^2}{sx_0}\right)^{\frac{1}{3}}(sx_0\cos^2\theta - 4x_0 + x)\,, \tag{12}$$

with solutions:

$$E(\eta) = D_1Ai(-\eta) + D_2Bi(-\eta)\,, \tag{13}$$

where D_i's are another integration constants.

4 The anomalous transmission

Consider the dielectric permittivity profile according to the geometry of Fig. 1. The field components in the plasma region are given by Eqs. (11) and (13) for regions $\mathbf{0} < \boldsymbol{x} < \mathbf{2}\boldsymbol{x_0}$ and $\mathbf{2}\boldsymbol{x_0} < \boldsymbol{x} < \mathbf{4}\boldsymbol{x_0}$, respectively. Hence, the constant parameters $\boldsymbol{C_i}$'s and $\boldsymbol{D_i}$'s give the electric field amplitudes in these regions. The field component in the vacuum regions $\boldsymbol{x} < \mathbf{0}$ is given by:

$$E(x) = E_0e^{ixk_0\cos\theta} + Re^{-ixk_0\cos\theta}\,, \tag{14}$$

where E_0 and R denote the incident and the reflected field amplitudes, respectively. For the vacuum region, at the rear side of the slab, namely for $x > 4x_0$, the field component becomes:

$$E(x) = Te^{ixk_0\cos\theta}\,, \tag{15}$$

where T is the transmitted field amplitude. In order to find the amplitudes C_i's, D_i's, R, and T we apply the boundary conditions. The boundary conditions are the continuity of the electric field E and its derivative $\frac{dE}{dx}$ at all boundaries of Fig. 1. By matching the solutions at the boundaries, a system of six equations can be obtained for the unknown coefficients. By solving the equations, the field amplitudes in all regions can be determined in terms of the incident wave amplitude E_0. At the first boundary, located at $x = 0$ one finds:

$$\begin{cases} E_0 + R = & C_1 Ai(-\zeta^0) + C_2 Bi(-\zeta^0), \\ E_0 - R = & -i\alpha\left[C_1 \acute{A}i(-\zeta^0) + C_2 \acute{B}i(-\zeta^0)\right], \end{cases} \tag{16}$$

where the prime sign indicates the differentiation with respect to the argument of the function, also:

$$\alpha = \left[\cos\,\theta\,(sk_0x_0)^{1/3}\right]^{-1}, \tag{17}$$

$$\zeta^0 = \frac{1}{\alpha^2}, \tag{18}$$

where ζ^0 denotes the amount of ζ given in Eq.(9), at $x = 0$, namely $\zeta^0 = \zeta(x = 0)$. On the second boundary ($x = 2x_0$), we have:

$$\begin{cases} (C_1 - D_1)Ai(-\zeta^2) = & (D_2 - C_2)Bi(-\zeta^2), \\ (C_1 + D_1)\acute{A}i(-\zeta^2) = & -(D_2 + C_2)\acute{B}i(-\zeta^2), \end{cases} \tag{19}$$

where:

$$\zeta^2 = \left(\frac{k_0^2 x_0^2}{s}\right)^{1/3}(s\cos^2\theta - 2), \tag{20}$$

and $\zeta^2 = \zeta(x = 2x_0) = \eta(x = 2x_0)$. Also at the third boundary ($x = 4x_0$), the boundary conditions are:

$$\begin{cases} D_1 Ai(-\zeta^0) + D_2 Bi(-\zeta^0) = & Te^{4ik_0x_0\cos\theta}, \\ i\alpha\left[D_1 \acute{A}i(-\zeta^0) + D_2 \acute{B}i(-\zeta^0)\right] = & Te^{4ik_0x_0\cos\theta}, \end{cases} \tag{21}$$

here we also have $\eta(x = 4x_0) = \zeta^0$. By solving Eqs. (16), (19) and (21) the unknown coefficients C_i, D_i, T and, R can be found. The results are presented in the figures.

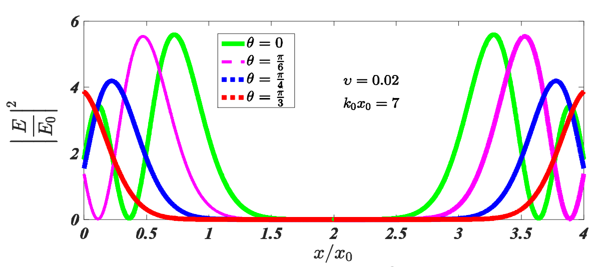

Fig. 2. The electric field distribution $\left|\frac{E}{E_0}\right|^2$ as a function of the position $\frac{x}{x_0}$ for four different values of the incident angle $\boldsymbol{\theta}$.

Figure 2 shows the electric field distribution $\left|\frac{E}{E_0}\right|^2$ as a function of $\frac{x}{x_0}$ for different incident angle θ. This figure indicates the formation of the surface waves at the boundaries of the plasma region. According to this figure, the fluctuations reduce and the wave amplitude decreases at higher incident angles. Therefore the transition properties of the slab are enhanced at small incident angles. This can also be seen in Fig. 3 where the transition amplitude $\left|\frac{T}{E_0}\right|^2$, from the entire slab of Fig. 1, is given as a function of the incident angel θ. In this figure, the reflection amplitude $\left|\frac{R}{E_0}\right|^2$ is also plotted, and it is obvious that $\left|\frac{T}{E_0}\right|^2 + \left|\frac{R}{E_0}\right|^2 = 1$.

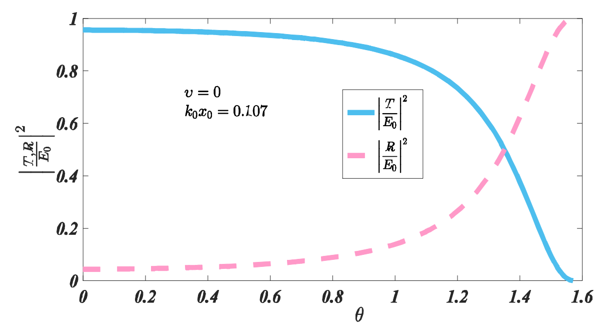

Fig. 3. The transition amplitude $\left|\frac{T}{E_0}\right|^2$ and the reflection amplitude $\left|\frac{R}{E_0}\right|^2$ versus the incident angle $\boldsymbol{\theta}$.

The effects of the value of k_0x_0 on the transition properties of the slab are shown in Fig. 4. This figure illustrates that the transition of the electromagnetic waves can occur at small values of k_0x_0 and specifically at $k_0x_0 < 0.2$. Therefore for the proposed model, the transition takes place at small widths of overdense plasma layer or metallic film, namely for $x_0 < \frac{0.2}{2\pi}\lambda$. For thicker layers, the reflection gets a higher share and is dominated. This is a feature of the anomalous light transition through an over-dense plasma layer or a metallic film. In fact, it has already been shown that the anomalous transition takes place for small thickness layers in the case of homogenous media [5].

The effect of collision frequency υ on the transmission amplitude can be seen in Fig. 5 where $\left|\frac{T}{E_0}\right|^2$ is plotted against θ for different values of υ . In this figure we have $k_0x_0 = 0.107$.

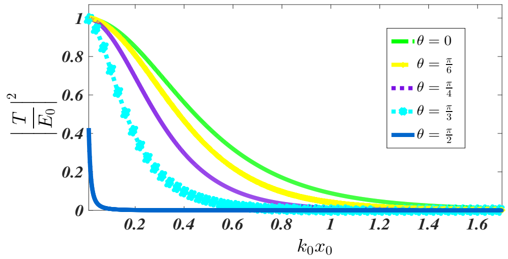

Fig. 4. The transition amplitude $\left|\frac{T}{E_0}\right|^2$ versus the value of $\boldsymbol{k_0 x_0}$ for different incident angels $\boldsymbol{\theta}$.

According to Fig. 5, by increasing the collision effects, the transmission and also the reflection reduce.

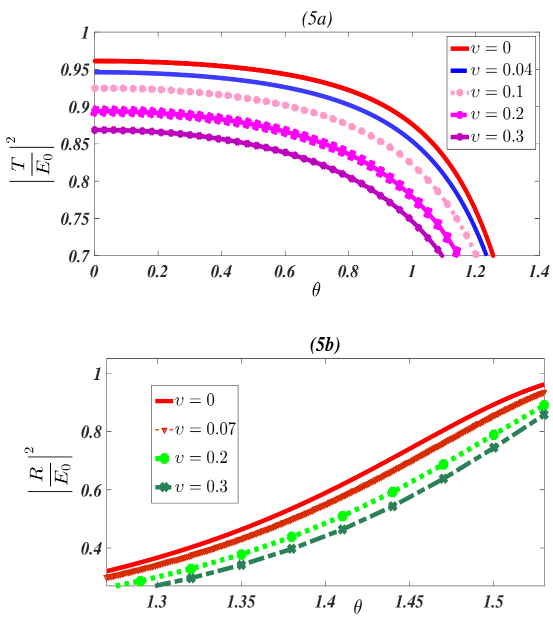

Fig. 5. The transition amplitude $\left|\frac{T}{E_0}\right|^2$ and the reflection amplitude $\left|\frac{R}{E_0}\right|^2$ versus the incident angle $\boldsymbol{\theta}$, respectively (5.a) and (5.b). Different collision frequencies $\boldsymbol{\upsilon}$ are applied.

5 Conclusions

In this study, we studied the anomalous transmission of light through an inhomogeneous over-dense plasma layer or a metallic film. It has already been shown that this anomalous transmission can be obtained for the homogenous medium through a mechanism of the simultaneous excitation of coupled surface waves. Through this mechanism, a homogenous dense plasma with $\varepsilon < 0$, as a left-handed material, is considered to be sandwiched between two diffracting layers such as grating and dielectric layers with $\varepsilon > 0$.

Here, we considered an inhomogeneous plasma layer with linear density which gradually becomes over-dense [23]. In this way, the effective dielectric permittivity reduces linearly to negative values (ultimately $\varepsilon = -1$) from the positive values (initially from $\varepsilon = 1$). In order to provide the conditions for the excitation of coupled surface waves, we also considered a plasma medium placed adjacent to the first one, in which the dielectric permittivity from the negative values acquires the positive values. In previous studies, we examined the conditions for the excitation of the surface waves for the structure.

Here, continued the previous studies, [23], we obtained the conditions for the transparency of the entire slab. We observed that the anomalous transmission only can be achieved for the incident electromagnetic waves with certain frequencies. In fact, the characteristic parameter of the thickness of the slab, namely x_0 must fulfill the condition $x_0 < \frac{0.2}{2\pi}\lambda$. Therefore, in order to have high transparency, the slab thickness must be small with respect to the wave length of the incident wave. The requirement of this condition has already been seen for the anomalous transmission of the homogeneous over-dense plasma [1] and [19]. This characteristic helps us to construct sub-wavelength super lenses with minimal chromatic aberration. Also, we observed that by increasing the incident angle, the wave transmission decreases and the reflection increases. Indeed, the anomalous transparency can be obtained for a range of incident angles, while in the case of homogeneous over-dense plasma the high transparency can be seen only at some specific resonant angles which are sharply distributed. This feature has significant effects on the transmission properties of the natural plasma mediums which gradually become dense and behave like LHM mediums. This particular property of the construction should be taken into account in designing more effective super lenses. We also consider the dissipation effects and showed that by increasing the collision frequency, the transmission and reflection amplitudes decrease. Therefore the dissipations, as expected, reduce the transmission ability of the structure. This effect also can be seen in the homogeneous case [23].

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